

Name: _____

Solutions

Math 1220-003 Quiz 11

July 26, 2018

You have until Monday, July 30th to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. Find the convergence set of the power series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n}$

Absolute ratio test:

$$\lim_{n \rightarrow \infty} \frac{|(x+2)^{n+1}/(n+1)|}{|(x+2)^n/n|} = \lim_{n \rightarrow \infty} \frac{|x+2|^{n+1}}{n+1} \cdot \frac{n}{|x+2|^n}$$

$$= \lim_{n \rightarrow \infty} |x+2| \cdot \frac{n}{n+1} = |x+2|$$

$$|x+2| < 1 \rightsquigarrow -1 < x+2 < 1 \rightsquigarrow -3 < x < -1$$

So converges when $-3 < x < -1$.

Check: if $x = -3$: $\sum \frac{(-1)^n}{n}$ converges (alternating harmonic series)

if $x = -1$: $\sum \frac{1^n}{n}$ diverges (harmonic series)

\Rightarrow convergence set is $[-3, -1)$

2. Find the first 4 terms of the Taylor series expansion of $\cos x$ at $x = \frac{\pi}{3}$.

$$i=0 \quad f(x) = \cos x \quad f\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$i=1 \quad f'(x) = -\sin x \quad f'\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$i=2 \quad f''(x) = -\cos x \quad f''\left(\frac{\pi}{3}\right) = -\frac{1}{2}$$

$$i=3 \quad f'''(x) = \sin x \quad f'''\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{1}{2} + \frac{-\frac{\sqrt{3}}{2}}{1!} \left(x - \frac{\pi}{3}\right) + \frac{-\frac{1}{2}}{2!} \left(x - \frac{\pi}{3}\right)^2 + \frac{\frac{\sqrt{3}}{2}}{3!} \left(x - \frac{\pi}{3}\right)^3$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{2} \left(x - \frac{\pi}{3}\right) - \frac{1}{4} \left(x - \frac{\pi}{3}\right)^2 + \frac{\sqrt{3}}{12} \left(x - \frac{\pi}{3}\right)^3$$