

Name: Solutions

Math 1220-003 Quiz 3

June 21, 2018

You have until the next class to complete this quiz. Make sure to write your name at the top of the quiz. This quiz is two questions, worth 20 points.

1. (10 points) Find exact values for the following:

(a) $\tan^{-1}(1)$

$$\tan^{-1}(1) = \theta \quad \text{such that} \quad \tan(\theta) = 1 \quad \text{and} \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$\Rightarrow \theta = \pi/4$$

(b) ~~$\sin(\sin^{-1}(\sqrt{2}-1)) = \sqrt{2}-1$~~

$$[\sin(\sin^{-1}(x)) = x \quad \text{always}]$$

$$[\sin^{-1}(\sin(x)) = x \quad \text{only if} \quad -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}]$$

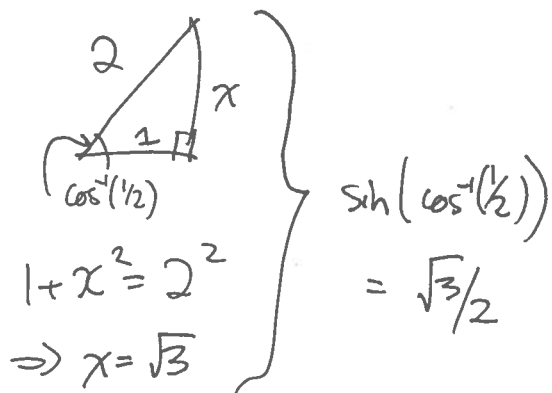
(c) $\tan^{-1}(\tan(\frac{5\pi}{6}))$

$$\tan(\frac{5\pi}{6}) = \tan(\frac{5\pi}{6} - \pi) = \tan(-\frac{\pi}{6})$$

$$\tan^{-1}(\tan(-\frac{\pi}{6})) = -\frac{\pi}{6}$$

$$[\tan^{-1}(\tan(x)) = x \quad \text{if} \quad -\frac{\pi}{2} < x < \frac{\pi}{2}]$$

(d) $\sin(2\cos^{-1}(\frac{1}{2})) = 2\sin(\cos^{-1}(\frac{1}{2})) - \cos(\cos^{-1}(\frac{1}{2})) = \frac{\sqrt{3}}{2}$



2. (10 points) Evaluate the following integral: $\int \frac{1}{\sqrt{3-2x-x^2}} dx$.

Complete the square: $-x^2 - 2x = -(x^2 + 2x) = -((x+1)^2 - 1)$

$$\int \frac{1}{\sqrt{3 - ((x+1)^2 - 1)}} dx = \int \frac{1}{\sqrt{4 - (x+1)^2}} dx$$

$$\begin{aligned} & \xrightarrow{u=x+1} \int \frac{1}{\sqrt{4-u^2}} du = \sin^{-1}\left(\frac{u}{2}\right) + C \\ & \begin{matrix} u = x+1 \\ du = dx \end{matrix} \end{aligned}$$

$$= \sin^{-1}\left(\frac{x+1}{2}\right) + C$$