1. Evaluate the following integrals:

(a) $\int_0^2 (x+4)(2x+1)\,dx$

$$= \int_0^2 (2x^2 + 9x + 4)\,dx$$

$$= \frac{2}{3}x^3 + \frac{9}{2}x^2 + 4x \Big|_0^2$$

$$= \frac{2}{3}(8) + \frac{9}{2}(4) + 4(2) = \boxed{31\tfrac{1}{3}}$$

(b) $\int \cos^4\theta \sin\theta\,d\theta$ $\quad u = \cos\theta$, $du = -\sin\theta\,d\theta$

$$\int \cos^4\theta \sin\theta\,d\theta = -\int u^4\,du = \frac{-1}{5}u^5 + C$$

$$= \boxed{\frac{-1}{5}\cos^5\theta + C}$$

(c) $\int x^3\sqrt{1+x^2}\,dx$ $\quad u = 1+x^2$, $du = 2x\,dx$

$x^2 = u - 1$

$$\frac{1}{2}\int \underbrace{2x}_{du} \cdot x^2\sqrt{1+x^2}\,\underbrace{dx}_{du}$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\,du = \frac{1}{2}\int (u^{3/2} - u^{1/2})\,du$$

$$= \frac{1}{2}\cdot\frac{2}{5}u^{5/2} - \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{1}{5}(1+x^2)^{5/2} - \frac{1}{3}(1+x^2)^{3/2} + C}$$

2(a) Write down two antiderivatives of $f(x) = \sec(\sqrt{1+x^2})$.

$$\int_0^x \sec(\sqrt{1+t^2})\,dt \quad , \quad \int_1^x \sec(\sqrt{1+t^2})\,dt$$

2(b) A wasp colony starts at week 0 with 20 wasps and increases thereafter at a rate of $w'(t)$ wasps per week. Write down an expression which represents the total number of wasps after 6 weeks.

$$20 + \int_0^6 w'(t)\,dt \text{ wasps}$$

2(c) Evaluate $\int_0^{\pi/8} \sec(2\theta)\tan(2\theta)\,d\theta$ $\quad u = 2\theta, \quad du = 2\,d\theta$

$$= \frac{1}{2}\int_0^{\pi/4} \sec(u)\tan(u)\,du$$

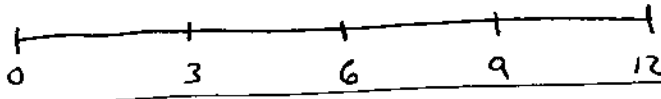
$$= \frac{1}{2}\sec(u)\Big|_0^{\pi/4} = \frac{1}{2}\sec(\pi/4) - \frac{1}{2}\sec(0)$$

$$= \frac{1}{2\cos(\pi/4)} - \frac{1}{2\cos(0)}$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{2}$$

3. Use four rectangles to estimate the area under the curve $y = 50 - \cos(x^2)$ between $x = 0$ and $x = 12$ as follows.

(a) Write down the sample points (choose left or right, and say which).



left : $x_1^* = 0$, $x_2^* = 3$, $x_3^* = 6$, $x_4^* = 9$

right : $x_1^* = 3$, $x_2^* = 6$, $x_3^* = 9$, $x_4^* = 12$

(b) Write out a complete numerical; expression for the area estimate. You do not need to evaluate the number.

$\Delta x = 3$

$$3(50 - \cos(3^2)) + 3(50 - \cos(6^2)) + 3(50 - \cos(9^2)) + 3(50 - \cos(12^2))$$

(using right endpoints)

(c) Express part (b) using sigma notation.

$$\sum_{i=1}^{4} 3(50 - \cos((3i)^2))$$

(right endpoints)

(d) Write down an expression for the area estimate given by using n rectangles (using sigma notation or not, as you like).

$\Delta x = 12/n$, $x_i^* = \frac{12i}{n}$

$$\sum_{i=1}^{n} \left(50 - \cos\left(\left(\frac{12i}{n}\right)^2\right)\right) \cdot \frac{12}{n}$$

4(a) Sketch carefully the region between the curves $y = x$ and $4x + y^2 = 12$.

4(b) Find the area of this region.

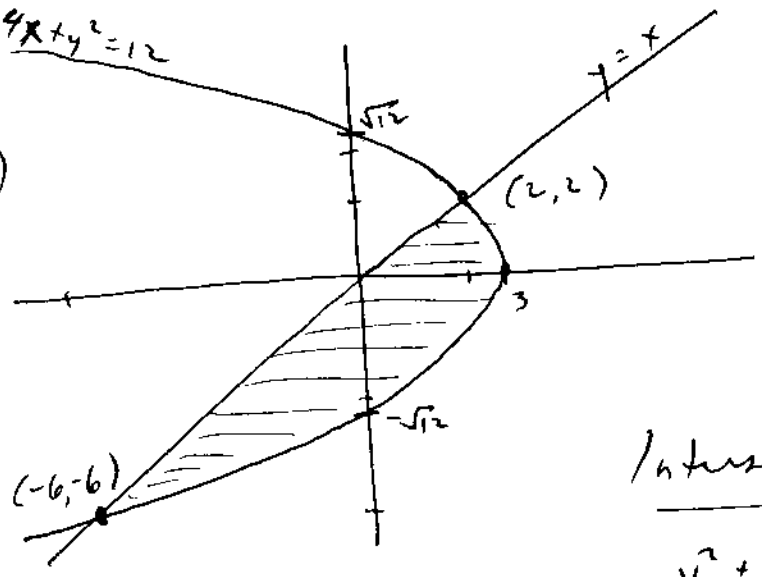
$4x+y^2=12$ ↓ a parabola.

$4(0)+y^2=12$

$y = \pm\sqrt{12}$ (y-intercepts)

$4x+(0)^2=12$, $x=3$ (x-intercept)

(a)



Intersection points: $4y+y^2=12$

$y^2+4y-12=0$

$(y+6)(y-2)=0$ $y=2,-6$

(b) Region bounded by $y=x$ on left, $4x+y^2=12$ on right.

$x = \frac{12-y^2}{4} = \frac{-y^2}{4}+3$

$$\text{Area} = \int_{-6}^{2} \left(\left(\frac{-y^2}{4}+3\right)-y\right) dy$$

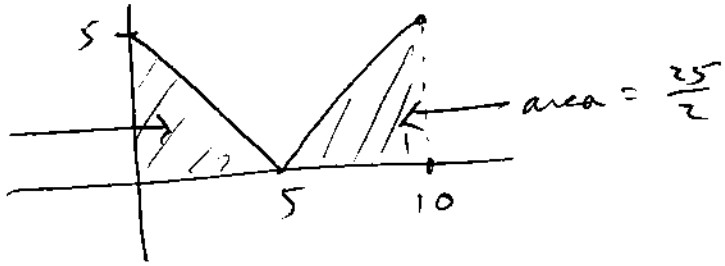
$$= \frac{-1}{12}y^3 + 3y - \frac{1}{2}y^2 \Big|_{-6}^{2}$$

$$= \left(\frac{-8}{12}+6-\frac{4}{2}\right) - \left(\frac{-(-6)^3}{12}+3(-6)-\frac{1}{2}(36)\right)$$

$$= \frac{-2}{3}+4-18+18+18 = \boxed{21\frac{1}{3}}$$

5(a) Evaluate $\int_0^{10} |x-5|\,dx$ by interpreting it in terms of areas. (Do not use the fundamental theorem.)

graph of $y = |x-5|$ is like $y = |x|$, shifted right by 5:



area 1 $25/2$

$$\int_0^{10} |x-5|\,dx = \text{area of two triangles} = \boxed{25}$$

5(b) Find the derivatives of

(i) $F(x) = \int_0^x \frac{t^2}{\cos t}\,dt$

$$\boxed{F'(x) = \frac{x^2}{\cos(x)}}$$

(ii) $\int_0^{5x^4} \frac{t^2}{\cos t}\,dt = F(5x^4)$ chain rule

has derivative $F'(5x^4) \cdot 20x^3$

$$= \boxed{\frac{(5x^4)^2}{\cos(5x^4)} \cdot 20x^3}$$