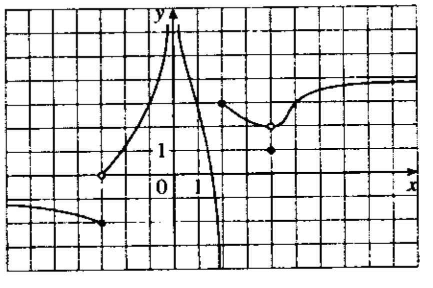
1(a) Let $f(x)$ be the function shown below.



(i) Find $\underbrace{\lim_{x\to -3^+} f(x)}_{= 0}$, $\underbrace{\lim_{x\to 0^+} f(x)}_{= +\infty}$, $\underbrace{\lim_{x\to 1^+} f(x)}_{= 3}$, $\underbrace{\lim_{x\to 2^+} f(x)}_{= 3}$, and $\underbrace{\lim_{x\to 4^+} f(x)}_{= 2}$.

(ii) Find the points at which f is not continuous.

at $x = -3, 0, 2, 4$

(iii) Find the points at which f is not continuous from the right.

at $x = -3, 0, 4$

1(b) Explain why the equation $3\cos x = 1 + 5x$ has a solution in the interval $(0, 1)$. What result are you using to know this?

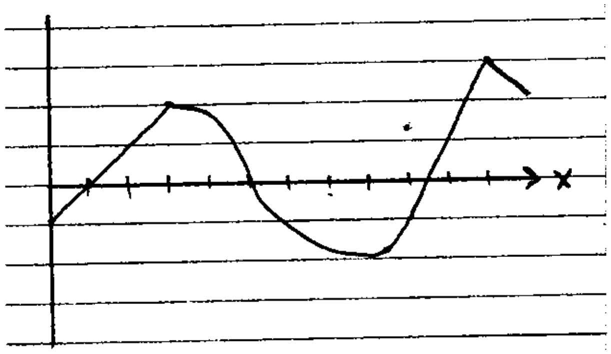
Consider $f(x) = 3\cos x - (1 + 5x)$.

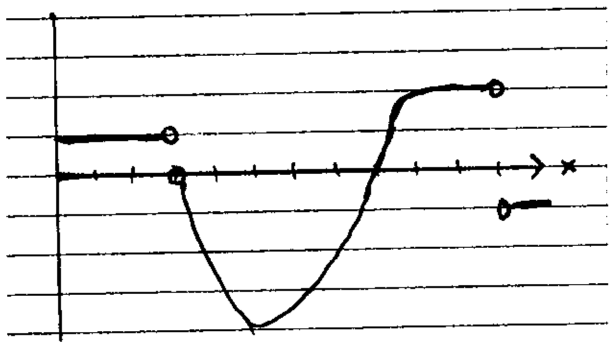
$f(0) = 3 - 1 = 2$

$f(1) = \underbrace{3\cos(1)}_{\text{at most } 3} - 6$, which is negative.

Since f is continuous, the Intermediate Value Theorem says that there is an x between 0 and 1 where $f(x) = 0$. But $f(x) = 0$ exactly when the equation is satisfied.

2(a) The graph of $f(x)$ is shown below. On the second set of axes, draw carefully the graph of the derivative $f'(x)$. Be as accurate as you can.





2(b) Find the limit

$$\lim_{h\to 0} \frac{(\pi+h)^2-\pi^2}{h}.$$

[Think!]

This is the definition of $f'(\pi)$, when $f(x) = x^2$.

Since $f'(x) = 2x$, the limit is $\boxed{2\pi}$.

(You can multiply out the top and find the limit directly, too.)

3. Find all points on the graph of the function

$$f(x) = 2\sin x + \sin^2 x, \qquad 0 \le x \le 4\pi$$

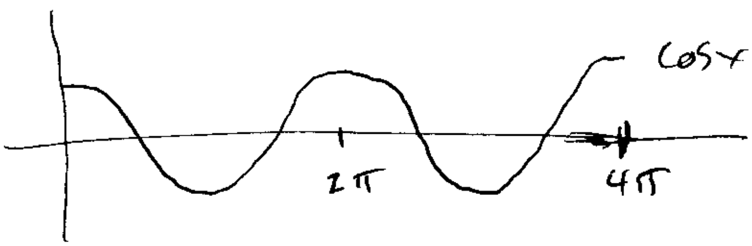
at which the tangent line is horizontal. These are the points where $f'(x) = 0$.

$$f'(x) = 2\cos x + 2\sin x \cos x$$

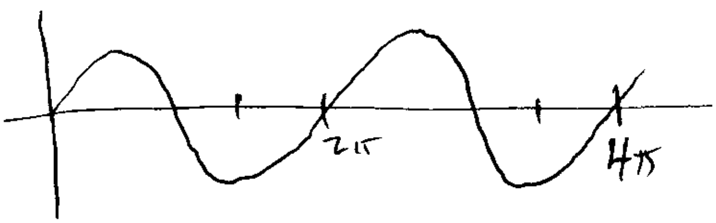
$$= 2\cos x(1 + \sin x).$$

$f'(x) = 0$ when $\cos x = 0$ or $\underbrace{1 + \sin x = 0}$

i.e. $\sin x = -1$.



On $[0, 4\pi]$, $\cos x = 0$ when $x = \underline{\pi/2,\ 3\pi/2,\ 5\pi/2,\ 7\pi/2}$.



On $[0, 4\pi]$, $\sin x = -1$ when $\underline{x = 3\pi/2,\ 7\pi/2}$.

finally, $f(\pi/2) = 2 + 1 = 3$

$f(3\pi/2) = -2 + 1 = -1$

$f(5\pi/2) = 3$, $f(7\pi/2) = -1$.

Points on graph:

$$\boxed{(\pi/2, 3),\ (3\pi/2, -1),\ (5\pi/2, 3),\ (7\pi/2, -1)}$$

4. Find the derivatives:

(a) $f(x) = \sqrt{\sqrt{\sqrt{x}}}$

chain rule:

$$\boxed{f'(x) = \frac{1}{2\sqrt{\sqrt{\sqrt{x}}}} \cdot \frac{1}{2\sqrt{\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}}}$$

or, notice that $f(x) = x^{\frac{1}{8}}$

$$\boxed{\text{so } f'(x) = \frac{1}{8} x^{-7/8}}$$

(these are the same)

(b) $f(x) = \tan\sqrt{1-x}$

$$\boxed{f'(x) = \sec^2\left(\sqrt{1-x}\right) \cdot \frac{1}{2\sqrt{1-x}} \cdot (-1)}$$

(c) $f(x) = x^3 \sin x$

$$\boxed{f'(x) = x^3 \cos x + 3x^2 \sin x}$$

(d) $f(x) = \dfrac{\sqrt{x}}{x+1}$

$$\boxed{f'(x) = \frac{(x+1)\frac{1}{2\sqrt{x}} - \sqrt{x}}{(x+1)^2}}$$

5. The equation of the motion of a particle is $s(t) = t^3 - 3t$, where s is in meters and t is in seconds.

   (i) Find the velocity and acceleration functions.

   (ii) Find the acceleration after 2 seconds.

   (iii) Find the acceleration when the velocity is 0.

(i) $\boxed{\begin{aligned} v(t) &= 3t^2 - 3 \\ a(t) &= 6t \end{aligned}}$

(ii) $\boxed{a(2) = 12 \ m/sec^2}$

(iii) $v = 0$ when $3t^2 - 3 = 0$

$t^2 = 1$

$t = \pm 1$

$\boxed{\begin{aligned} a(-1) &= -6 \ m/sec^2 \\ a(1) &= 6 \ m/sec^2 \end{aligned}}$