

1. Find the equation of the tangent line to the curve $x^2 + 4xy + y^2 = 13$ at the point $(2, 1)$.

take $\frac{d}{dx}$ of both sides:

$$\frac{d}{dx}(x^2 + 4xy + y^2) = \frac{d}{dx}(13)$$

$$2x + (4x \frac{dy}{dx} + 4y) + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(4x + 2y) = -2x - 4y$$

$$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$$

put in $(2, 1)$:

$$\frac{dy}{dx}(2, 1) = \frac{-4 - 4}{8 + 2} = \frac{-8}{10} = -\frac{4}{5}$$

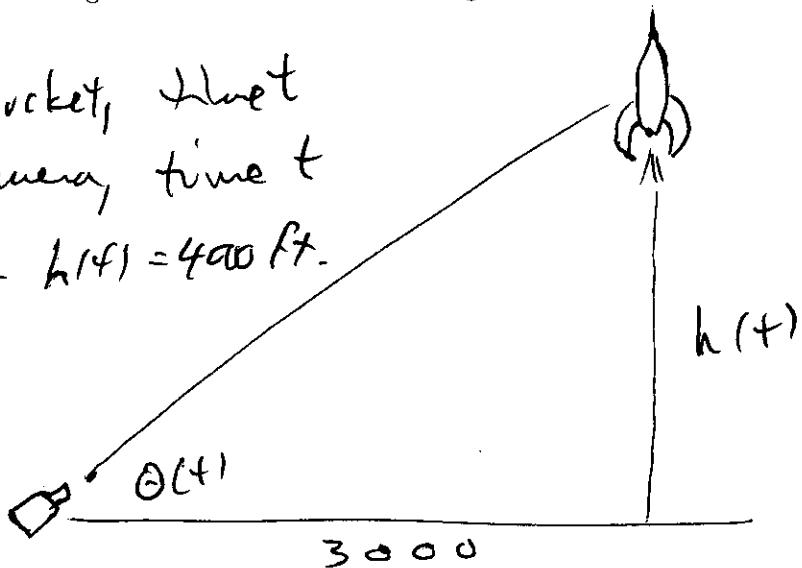
point-slope formula:

$$y - 1 = -\frac{4}{5}(x - 2)$$

$$(or \quad y = -\frac{4}{5}x + \frac{13}{5})$$

2. A camera is mounted on the ground 3000 ft. away from the launch pad of a rocket. It stays pointed at the rocket as the rocket takes off and rises. When the rocket is 4000 ft. above the ground, it is rising at a rate of 880 ft. per second. How fast is the angle the camera makes with the ground increasing at that instant? [Start with a picture!]

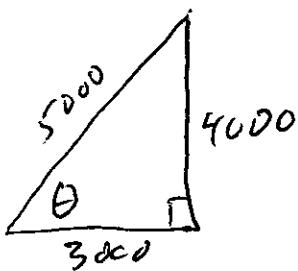
let $h(t)$ = height of rocket, time t
 $\theta(t)$ = angle of camera, time t
we want: $\frac{d\theta}{dt}$ when $h(t) = 4000$ ft.



relation: $\tan \theta = \frac{h}{3000}$

take $\frac{d}{dt}$: $\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{3000} \frac{dh}{dt}$

need to find $\sec \theta$ when $h(t) = 4000$ ft. 880 ft/sec .



$$\sec \theta = \frac{1}{\cos \theta} = \frac{5000}{3000} = \frac{5}{3}$$

we get: $(\frac{5}{3})^2 \frac{d\theta}{dt} = \frac{880}{3000}$

$$\frac{d\theta}{dt} = \frac{880 \cdot 9}{3000 \cdot 25} \text{ radians/sec}$$

3(a) Find the absolute maximum, and where it occurs, for $f(x) = 12 + 4x - x^2$ on the interval $[0, 5]$.

critical numbers?

$$f'(x) = 4 - 2x = 0$$

$$4 = 2x, \quad x = \underline{2}$$

check f at endpoints and critical numbers:

$$f(0) = 12$$

$$f(2) = 12 + 8 - 4 = 16$$

$$f(5) = 12 + 20 - 25 = 7.$$

Absolute maximum is 16, at $x=2$.

(b) Use the Mean Value Theorem (or Rolle's Theorem) to explain why $f(x) = 2x + \cos x$ has at most one root.

$f'(x) = 2 - \sin x$, which is always positive, because $\sin x$ is in $[-1, 1]$.

If there are two roots, say at a and b , then $f(a) = f(b) = 0$, so

$$\frac{f(b) - f(a)}{b - a} = 0. \text{ But MVT says}$$

that there is a c with $f'(c) = \frac{f(b) - f(a)}{b - a}$.

$$\text{i.e. } f'(c) = 0.$$

But this does not happen, so f cannot have two (or more) roots.

4. Consider the function $f(x) = \frac{x^2}{x^2 + 3}$.

(i) What kind of symmetry does f have?

(ii) What are the critical numbers? On what intervals is f increasing/decreasing?

(iii) On what intervals is the graph of f concave up/concave down?

(iv) Are there any asymptotes?

$$(i) f(-x) = \frac{(-x)^2}{(-x)^2 + 3} = \frac{x^2}{x^2 + 3} = f(x) \text{ so } \\ \boxed{f \text{ is even}}$$

$$(ii) f'(x) = \frac{(x^2+3) \cdot 2x - x^2(2x)}{(x^2+3)^2} = \frac{6x}{(x^2+3)^2}$$

$f'(x) = 0$. when $\boxed{x=0}$ only.
 ↑ critical numbers.

Also, denominator is > 0 , numerator is $\frac{+0}{+0}$

so $\boxed{f \text{ is increasing on } (0, \infty) \\ \text{decreasing on } (-\infty, 0)}$

$$(iii) f''(x) = \frac{(x^2+3)^2 \cdot 6 - 6x \cdot 2(x^2+3)(2x)}{(x^2+3)^4}$$

$$= \frac{(x^2+3) \cdot 6 - 24x^2}{(x^2+3)^3} = \frac{-18x^2 + 18}{(x^2+3)^3}$$

$$= \frac{-18(x^2-1)}{(x^2+3)^3} \quad = \text{zero when } x^2-1=0, \\ \text{i.e. when } x = \pm 1.$$

(over) \rightarrow

4 (cont)

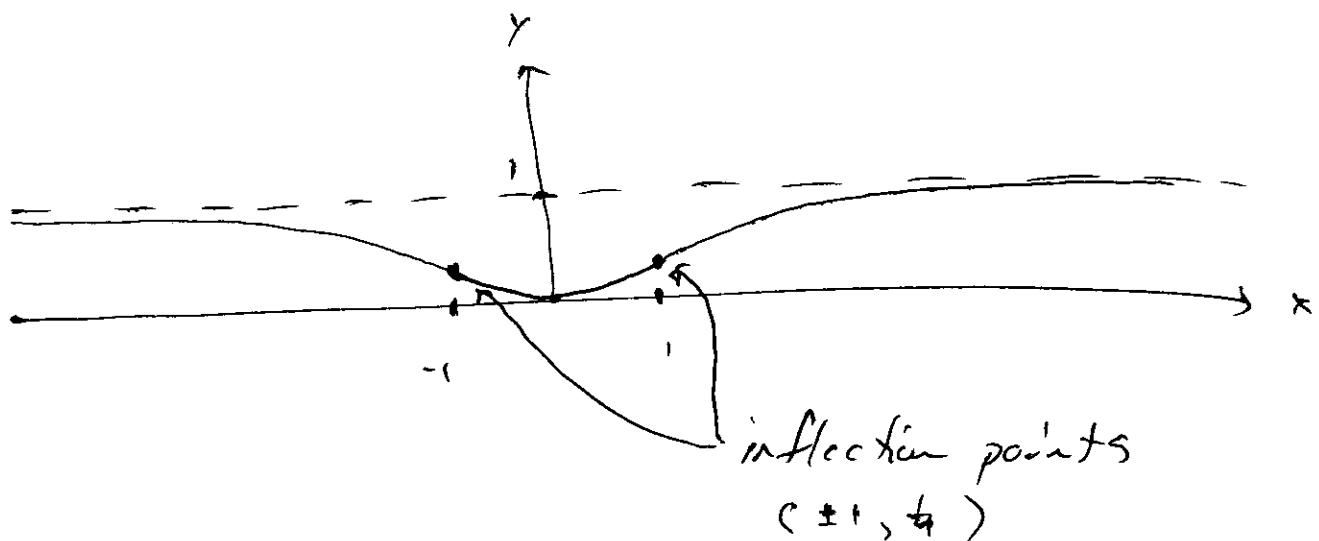
also, f'' is negative when $x^2 > 1$, positive when $x^2 < 1$

so, f is concave up on $(-1, 1)$
concave down on $(-\infty, -1)$ and $(1, \infty)$

$$(iv) \lim_{x \rightarrow \pm\infty} \frac{x^2}{x^4 + 3} = \lim_{x \rightarrow \pm\infty} \frac{1}{1 + 3/x^2} = \lim_{x \rightarrow \pm\infty} \frac{1}{1} = 1$$

so horizontal asymptote $y = 1$

(v) Carefully draw the graph of $f(x)$, indicating the features found in parts (i)–(iv).



5. Find the linearization of $f(x) = x^4$ at $x = 2$. Then, estimate $(1.999)^4$.

Linearization at a is $L(x) = f(a) + f'(a)(x-a)$

$$f'(x) = 4x^3$$

$$f(2) = 16, \quad f'(2) = 32 \quad \text{so}$$

$$\boxed{L(x) = 16 + 32(x-2)}$$

To estimate $(1.999)^4$, we say

$$(1.999)^4 \approx L(1.999)$$

$$= 16 + 32(1.999 - 2)$$

$$= 16 + 32(-.001)$$

$$= 16 - .032$$

$$= \boxed{15.968}$$