

1. Compute the indefinite integrals:

$$(a) \int (4-2x)^5 dx \quad u = 4-2x, \quad du = -2dx$$

$$\int \frac{1}{2} u^5 du = \frac{1}{2} \cdot \frac{1}{6} u^6 + C$$

$$= \boxed{\frac{-1}{12} (4-2x)^6 + C}$$

$$(b) \int \sin(\pi t) \cos(\pi t) dt \quad u = \sin(\pi t), \quad du = \pi \cos(\pi t) dt$$

$$\int \frac{1}{\pi} u du = \frac{1}{2\pi} u^2 + C$$

$$= \boxed{\left[\frac{1}{2\pi} \sin^2(\pi t) + C \right]}$$

($u = \cos(\pi t)$ also works; you get $\frac{-1}{2\pi} \cos^2(\pi t) + C$
which is the same family)

$$(c) \int x^5 \sqrt{1+x^2} dx \quad u = 1+x^2, \quad du = 2x dx$$

$$\int \frac{1}{2} \cdot 2x \cdot x^2 \cdot x^2 \cdot \sqrt{1+x^2} dx = \int \frac{1}{2} (u-1)^2 \sqrt{u} du$$

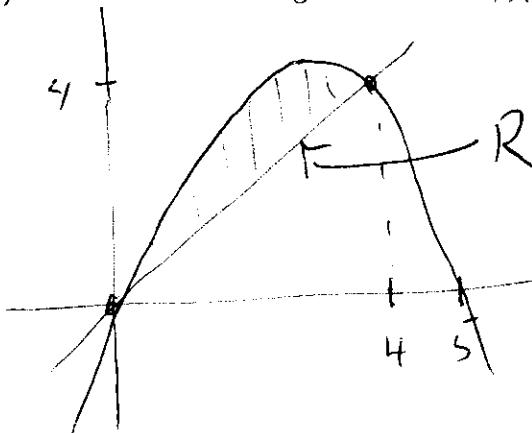
$$= \frac{1}{2} \int (u^2 - 2u + 1) u^{1/2} du = \frac{1}{2} \int (u^{5/2} - 2u^{3/2} + u^{1/2}) du$$

$$= \frac{1}{2} \left[\frac{2}{7} u^{7/2} - \frac{4}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right] + C$$

$$= \boxed{\left[\frac{1}{7} (1+x^2)^{7/2} - \frac{4}{5} (1+x^2)^{5/2} + \frac{2}{3} (1+x^2)^{3/2} + C \right]}$$

2. Consider the region R enclosed by the curves $y = x(5 - x)$ and $y = x$.

- (a) Draw the region carefully.
 (b) Find the area of this region.



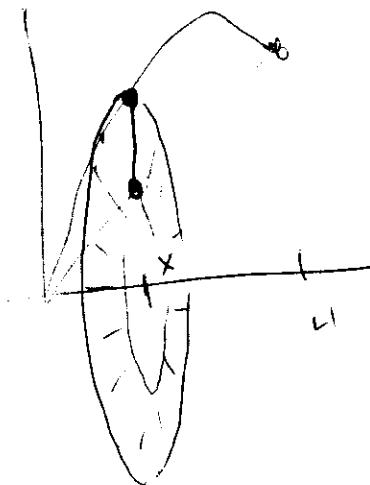
intersect's?

$$\begin{aligned}x &= 5x - x^2 \\0 &= (4 - x)x \\x &= 0, 4\end{aligned}$$

$$\begin{aligned}y &= (5-x)x \text{ intersects:} \\x &= 0, 5\end{aligned}$$

$$\begin{aligned}\text{Area} &= \int_0^4 (5x - x^2 - x) dx = \int_0^4 (4x - x^2) dx \\&= 2x^2 - \frac{1}{3}x^3 \Big|_0^4 = 32 - \frac{64}{3} \\&= \boxed{\frac{32}{3}}\end{aligned}$$

- (c) Consider the solid of revolution obtained by rotating the region R about the x -axis. Write down a definite integral which represents the volume of this solid.



$$\begin{aligned}A(x) &= \text{area of washer at } x \\&= \pi (x(5-x))^2 - \pi (x)^2\end{aligned}$$

$$\boxed{\text{Volume} = \int_0^4 (\pi (x(5-x))^2 - \pi (x)^2) dx}$$

- 3(a) If $r(t)$ is the rate at which water flows into a lake, in gallons per day, what does $\int_0^{50} r(t) dt$ represent? Be as specific as you can.

$\int_0^{50} r(t) dt$ is the total number of gallons of water that flowed into the lake during the first 50 days.

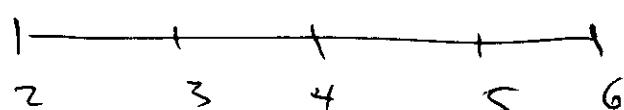
- (b) Find the derivatives of $g(x) = \int_0^x (1 - v^2)^5 dv$ and $h(x) = \int_0^{\cos(x)} (1 - v^2)^5 dv$.

$$\boxed{g'(x) = (1 - x^2)^5}$$

$$h(x) = g'(\cos(x)) \text{ so } h'(x) = g'(\cos(x))(-\sin x)$$

$$\boxed{h'(x) = (1 - (\cos x)^2)^5 (-\sin x)}$$

- (c) Write down a Riemann sum for $f(x) = \sqrt{\cos(x)}$ over the interval $2 \leq x \leq 6$ for $n = 4$, using right endpoints. [Start by writing down the sample points.] Either use sigma notation, or write the sum out completely.



four sub-intervals

Sample points: 3, 4, 5, 6

$$\Delta x = 1$$

Riemann Sum:

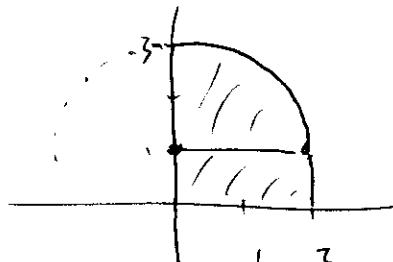
$$\left(\sum_{i=1}^4 \sqrt{\cos(i+2)} \cdot 1 \right)$$

$$\boxed{\sqrt{\cos(3)} \cdot 1 + \sqrt{\cos(4)} \cdot 1 + \sqrt{\cos(5)} \cdot 1 + \sqrt{\cos(6)} \cdot 1}$$

4(a) Evaluate $\int_0^2 (1 + \sqrt{4 - x^2}) dx$ by interpreting in terms of areas.

$y = \sqrt{4-x^2}$ is a semicircle of radius 2 centered at origin, so

$y = 1 + \sqrt{4-x^2}$ is a semicircle shifted up by 1:



Integral gives area shaded

$$= \frac{\pi(2)^2}{4} + 2 = \boxed{\pi + 2}$$

4(b) Evaluate $\int_{\pi^2}^{4\pi^2} \frac{1}{\sqrt{x}} \sin(\sqrt{x}) dx$.

$$u = \sqrt{x}, \quad du = \frac{1}{2\sqrt{x}} dx$$

$$2 \int_{\pi}^{2\pi} \sin(u) du = -2 \cos(u) \Big|_{\pi}^{2\pi}$$

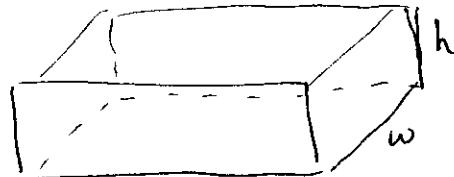
$$= -2 \cos(2\pi) + 2 \cos(\pi)$$

$$= -2 - 2$$

$$= \boxed{-4}$$

5. A cardboard box with an open top has its length equal to twice its width, and has volume 10 m^3 . The material for the sides costs \$6 per square meter, and the material for the bottom costs \$10 per square meter.

- (a) Find the cost of the box, as a function of the width. What is the domain of your cost function?
 (b) Find the dimensions that minimize the cost of the box.



$$\text{cost of bottom} = 10 \cdot 2w^2 = 20w^2$$

$$\text{cost of sides} = 6(4wh + 2wh) = 36wh$$

↑
 front+back ↗ left + right

what is h ?

$$10 = 2w^2h \rightarrow h = \frac{5}{w^2}$$

$$C(w) = 20w^2 + 36w \cdot \frac{5}{w^2}$$

$$\boxed{C(w) = 20w^2 + \frac{180}{w}, \quad w > 0}$$

$$C'(w) = 40w - \frac{180}{w^2} = 0$$

$$40w^3 = 180$$

$$w^3 = \frac{9}{2}$$

$$w = \sqrt[3]{\frac{9}{2}}$$

one crit. point, = minimum

$$\underline{C'(w_0)} + \underline{C'(w_0)}$$

$$\sqrt[3]{\frac{9}{2}}$$

$$h = \frac{5}{(\sqrt[3]{\frac{9}{2}})^2} = 5 \cdot (\frac{9}{2})^{-\frac{2}{3}}$$

best dimensions:

$$\boxed{\sqrt[3]{\frac{9}{2}} \text{ m wide} \times 2\sqrt[3]{\frac{9}{2}} \text{ m long} \times 5 \cdot (\frac{9}{2})^{-\frac{2}{3}} \text{ m high}}$$