1. Compute the following indefinite and definite integrals:

(a) $\int (t^2 - 1)^2 \, dt$

$$= \int (t^4 - 2t^2 + 1)\, dt$$

$$= \boxed{\frac{1}{5}t^5 - \frac{2}{3}t^3 + t + C}$$

(b) $\int \cos(x)\cos(\sin(x))\, dx$

$u = \sin(x)$, $du = \cos(x)\, dx$

$$= \int \cos(u)\, du$$

$$= \sin(u) + C = \boxed{\sin(\sin(x)) + C}$$

(c) $\int_0^1 (1-x)^9 \, dx$

$u = 1 - x$, $du = -dx$

$$= -\int_1^0 u^9 \, du = \int_0^1 u^9 \, du$$

$$= \frac{1}{10}u^{10}\Big|_0^1 = \frac{1}{10}(1)^{10} - \frac{1}{10}(0)^{10}$$

$$= \boxed{\frac{1}{10}}$$

2. Consider the region R enclosed by the curves $y = x(4-x)$ and $y = x$.

(a) Find where the curves intersect, and draw the region carefully.

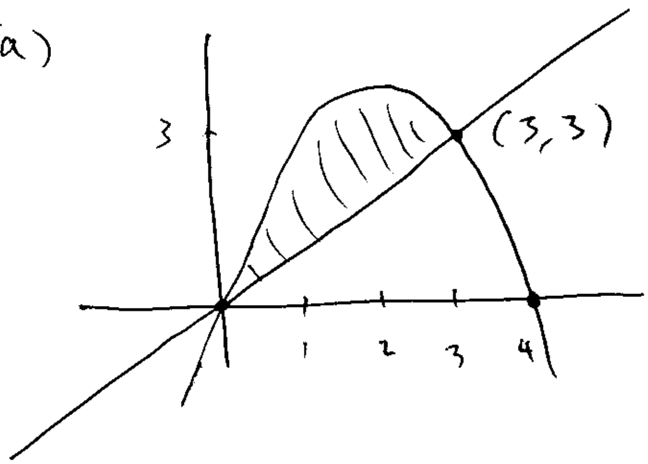
(b) Find the area of this region.

intersect when $4x - x^2 = x$

$3x - x^2 = 0$

$(3-x)x = 0 \quad x = 0, 3$

(a)



intersect at $(0,0)$ and $(3,3)$

(b) Area $= \int_0^3 (x(4-x) - x)\,dx$

$= \int_0^3 (3x - x^2)\,dx$

$= \frac{3}{2}x^2 - \frac{1}{3}x^3 \Big|_0^3$

$= \left(\frac{3}{2}(3)^2 - \frac{1}{3}(3)^3\right) - \left(\frac{3}{2}(0)^2 - \frac{1}{3}(0)^3\right)$

$= \frac{27}{2} - 9 = \boxed{\frac{9}{2}}$

3(a) If $r(t)$ is the rate at which water flows into a lake, in gallons per day, what does $\int_0^{25} r(t)\,dt$ represent? Be as specific as you can.

It is the increase, in gallons, of the amount of water in the lake, from day 0 to day 25.

(b) Find the derivatives of $g(x) = \int_4^x \sin(t^2)\,dt$ and $h(x) = \int_4^{3x^2} \sin(t^2)\,dt$.

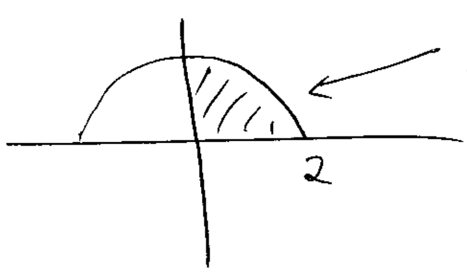
$$\boxed{g'(x) = \sin(x^2)}$$

$$h(x) = g(3x^2) \quad \text{so} \quad h'(x) = g'(3x^2)\cdot 6x$$

$$\boxed{h'(x) = \sin\left((3x^2)^2\right)\cdot 6x}$$

(c) Evaluate $\int_0^2 \sqrt{4 - x^2}\,dx$ by interpreting in terms of area. [What is the curve?]

$y = \sqrt{4-x^2}$ is the circle of radius 2, centered at the origin.



integral is this area

$= \frac{1}{4} \times$ area of circle of radius 2

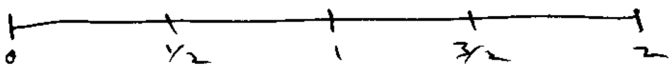
$= \frac{1}{4}\pi(2)^2 = \boxed{\pi}$

4. Use four rectangles to estimate the area under the curve $y = \sin(x^2)$ between $x = 0$ and $x = 2$ as follows:

(a) Write down the sample points (choose left or right, and say which).

$\boxed{\text{left endpoints: } x_1^* = 0,\ x_2^* = \tfrac{1}{2},\ x_3^* = 1,\ x_4^* = 3/2}$

or: $\boxed{\text{right endpoints: } x_1^* = 1/2,\ x_2^* = 1,\ x_3^* = 3/2,\ x_4^* = 2}$



(b) Write out a complete expression for the estimate.

$$\boxed{\tfrac{1}{2}\sin(0^2) + \tfrac{1}{2}\sin((\tfrac{1}{2})^2) + \tfrac{1}{2}\sin(1^2) + \tfrac{1}{2}\sin((3/2)^2)}$$

or

$$\boxed{\tfrac{1}{2}\sin((\tfrac{1}{2})^2) + \tfrac{1}{2}\sin(1^2) + \tfrac{1}{2}\sin((3/2)^2) + \tfrac{1}{2}\sin(2^2)}$$

(c) Express part (b) using sigma notation.

$$\boxed{\sum_{i=1}^{4} \tfrac{1}{2}\sin\left(\left(\tfrac{i-1}{2}\right)^2\right)} \quad \text{or} \quad \boxed{\sum_{i=1}^{4} \tfrac{1}{2}\sin\left(\left(\tfrac{i}{2}\right)^2\right)}$$

(d) Write down an expression for the area estimate given by using n rectangles (use sigma notation if you like).

$\Delta x = \frac{2}{n}$, $x_i^* = \frac{2}{n}(i-1)$ (left)

or $\frac{2}{n} i$ (right)

$$\boxed{\sum_{i=1}^{n} \frac{2}{n}\sin\left(\left(\frac{2}{n}i\right)^2\right)}$$

(right)

(for left, use $i-1$ in place of i)