# LINEAR ALGEBRA

## HOMEWORK - 2

1.3

#8 Find all values of $x$ so that $u \cdot u = 50$ where $u = \begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix}$

Sol: $u \cdot u = \begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ 3 \\ 4 \end{bmatrix} = x^2 + 9 + 16 = x^2 + 25$

$\Rightarrow x^2 + 25 = 50$

$x^2 = 50 - 25 = 25$

$\Rightarrow x = \pm 5$.

#12

a) $DA + B = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} -1 & 4 & 1 \\ 12 & 9 & 26 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}$

Can't add.

b) $EC = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & 1 \\ 12 & 5 & 17 \\ 19 & 0 & 22 \end{bmatrix}$

(c) $CE = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix}$

$CE = \begin{bmatrix} 15 & -7 & 14 \\ 23 & -5 & 29 \\ 13 & -1 & 17 \end{bmatrix}$

(d) $EB + F = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9 & 6 \\ 14 & 9 \\ 10 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$

$= \begin{bmatrix} 8 & 8 \\ 14 & 13 \\ 13 & 9 \end{bmatrix}$

(e) $FC + D$

Can't multiply $FC$ as

$F : 3 \times \underline{2}$ & $C : \underline{3} \times 3$

#22

(a) Second column : $A\, col_2(B)$

$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 0 \\ 13 \end{bmatrix}$

(b) $4^{th}$ column : $A \, col_4(B)$

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 4 \\ 4 & -2 & 3 \\ 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 3 \\ 13 \end{bmatrix}$$

1.4

#8 $$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

a) $$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

b) $$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(3\theta) & \sin(3\theta) \\ -\sin(3\theta) & \cos(3\theta) \end{bmatrix}$$

(use trignometric identities
$\sin(A \pm B)$
$\cos(A \pm B)$.

(c) In general,

$$A^K = \begin{bmatrix} \cos K\theta & \sin K\theta \\ -\sin K\theta & \cos K\theta \end{bmatrix}$$

(d) Proof of (c). Lets prove by induction.

If $K=2$, then we know that the result is true from part (a) for $A^2$.

Let's assume that the result is true for $K$.

We need to show that the result is true for $K+1$.

So,

$$A^{K+1} = A^K A$$

$$A^{K+1} = \begin{bmatrix} \cos(K\theta) & \sin(K\theta) \\ -\sin(K\theta) & \cos(K\theta) \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^{K+1} = \begin{bmatrix} \cos(K\theta+\theta) & \sin(K\theta+\theta) \\ -\sin(K\theta+\theta) & \cos(K\theta+\theta) \end{bmatrix}$$ (use trignometric identities).

$$A^{K+1} = \begin{bmatrix} \cos(K+1)\theta & \sin(K+1)\theta \\ -\sin(K+1)\theta & \cos(K+1)\theta \end{bmatrix}$$

The result is thus true for $K+1$

#10: Find two different $2\times 2$ matrices $A$ such that

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Sol: If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

then $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

If $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

then $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.