EXAM 3
Math 1914
10-24-14

Name Key

Instructions *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(10 points)* Give a definition of the definite integral of a function as a limit of Riemann sums. You should explain the meaning of the symbols you use in your definition.

Suppose $f(x)$ is a function defined for $a \le x \le b$. 1 Subdivide the interval $[a,b]$ into n 1 subintervals of equal length, Δx 1 and 1 for each choice of the number i from 1 to n, let x_i^* be any point in the i^{th} subinterval.

The definite integral of f on $[a,b]$ is defined to be

$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x,$$

2 2 2

provided the limit exists.

2. *(15 points)* Use a linear approximation (or differentials) to estimate $(2.007)^5$. Show your work.

Let $f(x) = x^5$ 1, and choose $x = 2.007$ 1 and $a = 2$. 1

Then $f'(x) = 5x^4$ 2. So $f(a) = 2^5 = 32$ 2, and

$f'(a) = 5 \cdot 2^4 = 5 \cdot 16 = 80$. 2

Then $f(x) \approx f(a) + f'(a)\cdot(x-a)$ 3, so

$(2.007)^5 \approx 32 + 80\,(0.007)$, or

$(2.007)^5 \approx 32.56$ 3

 Name ______________________

3. *(15 points)* If $f''(x) = 1 - \sin x$, $f'(0) = 5$, and $f(0) = 7$, find $f(x)$.

We have $f'(x) = \int (1 - \sin x)\,dx = x + \cos x + C$ (2) (2)

for some constant C. Since $f'(0) = 5$, then $0 + \cos 0 + C = 5$, (2)

so $1 + C = 5$, so $C = 4$. (1) Then $f'(x) = x + \cos x + 4$.

Hence $f(x) = \int (x + \cos x + 4)\,dx = \frac{x^2}{2} + \sin x + 4x + D$. (2) (2) (2)

Since $f(0) = 7$, then $0 + \sin 0 + 4 \cdot 0 + D = 7$, so $D = 7$, and (3)

$\boxed{f(x) = x^2/2 + \sin x + 4x + 7}$

4. *(15 points)* An object starts to move at time $t = 0$. When it starts moving, its velocity is 2 meters per second. Its acceleration, as a function of time, is $a = 5t$ meters per second per second. How far does the object move in 3 seconds?

The velocity v is $v = \int a\,dt = \int 5t\,dt = \frac{5t^2}{2} + C$. (2) (3)

Since $v = 2$ when $t = 0$, then $2 = 0 + C$, so $C = 2$,

so $v = \frac{5t^2}{2} + 2$. (2) Then the ~~displacement~~ position of

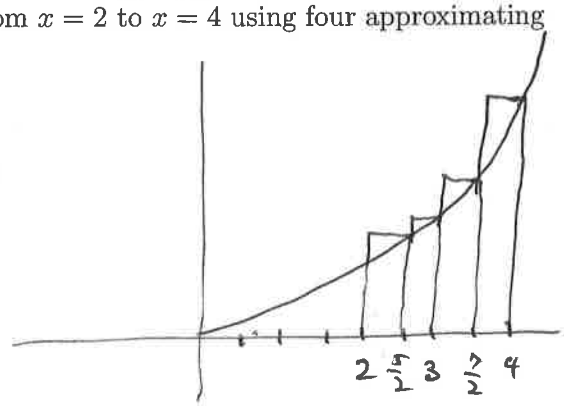
the object s is given by $s = \int v\,dt = \int \left(\frac{5t^2}{2} + 2\right)dt =$ (2)

$= \frac{5t^3}{6} + 2t + D$. (4) When $t = 3$, we have $s = \frac{5 \cdot 27}{6} + 6 + D$

so $s = \frac{45}{2} + 6 + D$. When $t = 0$, $s = D$. (2)

So distance moved is $\frac{45}{2} + 6 = 28\frac{1}{2}$. m

5. *(15 points)* Estimate the area under the graph of $f(x) = x^2$ from $x = 2$ to $x = 4$ using four approximating rectangles and right endpoints.



The heights of the four rectangles are $\left(\frac{5}{2}\right)^2, (3)^2, \left(\frac{7}{2}\right)^2$, and 4^2. (8)

Their bases have length $\frac{1}{2}$. (3)

So their total area is

$\frac{25}{4} \cdot \frac{1}{2} + 9 \cdot \frac{1}{2} + \frac{49}{4} \cdot \frac{1}{2} + 16 \cdot \frac{1}{2}$ (4)

$= \frac{1}{2}\left(\frac{25 + 36 + 49 + 64}{4}\right) = \frac{174}{8} = 21\frac{3}{4}$.

Name ______________________

6. *(10 points)* Consider the function $f(x) = \int_x^{\sin x} \frac{1}{\sqrt{1+t^3}}\, dt$

a. Fill in the blanks to obtain a different valid expression for $f(x)$:

$$f(x) = \int_0^{\sin x} \frac{1}{\sqrt{1+t^3}}\, dt - \int_0^{x} \frac{1}{\sqrt{1+t^3}}\, dt.$$

2

b. Find the derivative of $f(x)$ (Hint: you can use your answer to part **a**.)

Let $g(x) = \int_0^x \frac{1}{\sqrt{1+t^3}}\, dt$. By part 1 of The Fund. Th. of Calculus, 2

$g'(x) = \frac{1}{\sqrt{1+x^3}}$ 2. From part a, we have $f(x) = g(\sin x) - g(x)$.

So, by the Chain Rule,

$$f'(x) = g'(\sin x)\cdot \cos x - g'(x) = \boxed{\frac{1}{\sqrt{1+(\sin x)^3}}\cdot \cos x - \frac{1}{\sqrt{1+x^3}}}.$$

2 2

7. *(20 points)* Evaluate the definite integrals.

a. $\int_1^8 \frac{x^2+\sqrt[3]{x}}{x}\, dx = \int_1^8 \left(\frac{x^2}{x} + \frac{x^{1/3}}{x}\right) dx = \int_1^8 \left(x + x^{-2/3}\right) dx =$ 2 2

[10]

$$= \left[\frac{x^2}{2} + \frac{x^{1/3}}{(1/3)}\right]_1^8 = \left[\frac{x^2}{2} + 3\sqrt[3]{x}\right]_1^8$$

2 2 2

$$= \left(\frac{64}{2} + 3\cdot 2\right) - \left(\frac{1}{2} + 3\cdot 1\right) = \boxed{34\tfrac{1}{2}}$$

[10] **b.** $\int_0^{\pi/4} (\sec\theta - 1)(\sec\theta + 1)\, d\theta = \int_0^{\pi/4} (\sec^2\theta - 1)\, d\theta$ 2

$$= \left[\tan\theta - \theta\right]_0^{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right) - (\tan 0 - 0)$$

2 2 2

$$= \boxed{1 - \frac{\pi}{4}}$$ 2