

Quiz 2

Name: key

1. Find the derivatives of the following functions. You may use any rule of differentiation.

[12]

a) $y = \frac{4x}{x^2 + 4}$

By the quotient rule with $u = 4x$ and $v = x^2 + 4$,

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} = \frac{(x^2 + 4) \cdot 4 - 4x \cdot (2x)}{(x^2 + 4)^2}$$

b) $y = \left(x^3 + \frac{32}{x^3}\right)^2$. Let $u = x^3 + \frac{32}{x^3} = x^3 + 32x^{-3}$.

Then $\frac{du}{dx} = 3x^2 + 32(-3x^{-4})$, and since $y = u^2$, we

$$\text{have } \frac{dy}{dx} = 2u \frac{du}{dx} = 2\left(x^3 + \frac{32}{x^3}\right) (3x^2 + 32 \cdot (-3x^{-4}))$$

[8]

2. Find the equation of the tangent line to the graph of $f(x) = \frac{1}{x^2 + 1}$ at the point $(-1, \frac{1}{2})$. With $u = 1$ and $v = x^2 + 1$,

$$\frac{dy}{dx} = \frac{(x^2 + 1) \cdot 0 - 1 \cdot (2x)}{(x^2 + 1)^2}$$

Quotient rule, so at $x = -1$,

$$\frac{dy}{dx} = \frac{-2x}{(x^2 + 1)^2} = \frac{(-2)(-1)}{2^2} = \frac{1}{2}$$

The tangent line has slope $\frac{1}{2}$ and passes thru the point $(-1, \frac{1}{2})$, so it has the equation

$$\left(y - \frac{1}{2}\right) = \frac{1}{2}(x - (-1))$$

