Quiz 3

Name: ___key___

Please show all work for full credit.

1. Find the derivatives of the following functions. You may use any rule of differentiation. You don't need to simplify your answer.

a) $y = \dfrac{x}{(x^2+3)^{10}}$

[5]

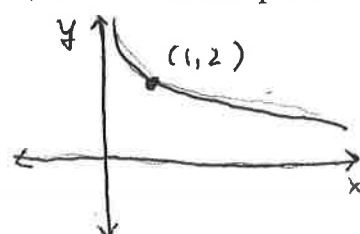$$\frac{dy}{dx} = \frac{(x^2+3)^{10} \cdot 1 - x \cdot 10(x^2+3)^9 \cdot 2x}{(x^2+3)^{20}}$$

[5] b) $y = \tan(\sin(\sqrt{x}))$

Here $y = f(g(x))$ where $f(x) = \tan x$
and $g(x) = \sin\sqrt{x}$.

So $f'(x) = \sec^2 x$, and $f'(g(x)) = \sec^2(\sin\sqrt{x})$.
To find $g'(x)$, write $g(x) = h(w(x))$, where $h(x) = \sin x$
and $w(x) = \sqrt{x}$. So $h'(x) = \cos x$ and $w'(x) = \frac{1}{2\sqrt{x}}$. So

$g'(x) = h'(w(x)) \cdot w'(x) = \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$. And $\boxed{f'(g(x)) \cdot g'(x) = \sec^2(\sin\sqrt{x}) \cdot \cos(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}}$

2. Find the equation of the tangent line to the graph of $xy^4 + 1 = 17$ at the point $(1,2)$.

(1,2)

$$\frac{d}{dx}\left(xy^4 + 1\right) = \frac{d}{dx}(17)$$

$$x \cdot \frac{d}{dx}(y^4) + 1 \cdot y^4 + 0 = 0$$

$$x \cdot 4y^3 \frac{dy}{dx} + y^4 = 0$$

$$4xy^3 \frac{dy}{dx} = -y^4$$

$$\frac{dy}{dx} = \frac{-y^4}{4xy^3} = \frac{-y}{4x}, \quad \text{at } (x,y)=(1,2), \quad \frac{dy}{dx} = \frac{-2}{4} = \frac{-1}{2}.$$

So the slope of the tangent line is $-\frac{1}{2}$, and
the equation is $(y-2) = -\frac{1}{2}(x-1)$.