Quiz 7

Name: key

[10] **1.** Evaluate $\int_0^1 \frac{x^2}{(1+x^3)^5}\,dx$.

Let $u = 1+x^3$ (1) $\quad x=0 \to u=1$ (1) $\quad x=1 \to u=2$

$du = 3x^2\,dx$ (1)

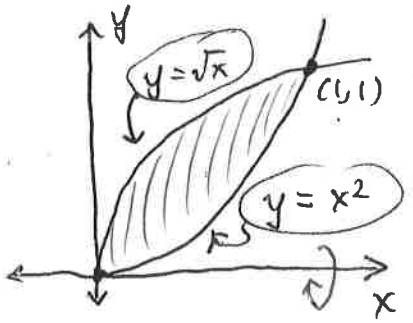
$dx = \frac{du}{3x^2}$ (1)

So $\int_{x=0}^{x=1} \frac{x^2}{(1+x^3)^5}\,dx = \int_{u=1}^{u=2} \frac{x^2}{u^5}\,\frac{du}{3x^2} = \frac{1}{3}\int_1^2 \frac{du}{u^5} =$ (1)

$= \frac{1}{3}\int_1^2 u^{-5}\,du = \frac{1}{3}\left[\frac{u^{-4}}{-4}\right]_1^2 = -\frac{1}{12}\left[\frac{1}{u^4}\right]_1^2 =$ (1) (1)

$= -\frac{1}{12}\left[\frac{1}{2^4} - \frac{1}{1^4}\right] = -\frac{1}{12}\left[\frac{1}{16} - 1\right] = -\frac{1}{12}\cdot\frac{-15}{16} = \frac{15}{12\cdot 16} = \boxed{5/64}$ (1)

[10] **2.** The shaded region lies between the graphs of $y = \sqrt{x}$ and $y = x^2$. Find the volume obtained by revolving it around the x-axis.



(1) (1) (1) (1)

$V = \pi\int_0^1 (\sqrt{x})^2\,dx - \pi\int_0^1 (x^2)^2\,dx$

(1) (1)

$= \pi\int_0^1 x\,dx - \pi\int_0^1 x^4\,dx$

(1) (1)

$= \pi\left[\frac{x^2}{2}\right]_0^1 - \pi\left[\frac{x^5}{5}\right]_0^1$

$= \pi\left[\frac{1}{2}\right] - \pi\left[\frac{1}{5}\right] = \boxed{\frac{3\pi}{10}}$

(1)