Quiz 5

Name: key

1. Use a linear approximation (or differentials) to estimate $\sqrt{49.01}$. Show your work.

Let $x = 49.01$, $f(x) = \sqrt{x}$, and $\tilde{x} = 49$. 3

Then $f(x) \cong f(\tilde{x}) + (x-\tilde{x})\cdot f'(\tilde{x})$ 3

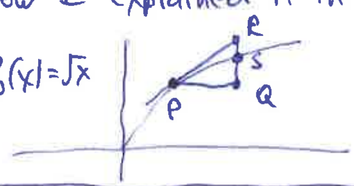
So $\sqrt{49.01} \cong \sqrt{49} + (0.01)\, f'(49)$.

Since $f'(x) = \frac{1}{2\sqrt{x}}$, Then $f'(49) = \frac{1}{2\sqrt{49}} = \frac{1}{14}$. 2

So $\sqrt{49.01} \cong 7 + (0.01)\cdot\frac{1}{14}$ 2 $= \boxed{7 + \frac{1}{1400}}$

(Here's how I explained it in class:

$f(x) = \sqrt{x}$



$P = (49, 7)$

$Q = (49.01, 7)$

$S = (49.01, \sqrt{49.01})$

$QS \cong QR = (\text{slope of } PR)\cdot PQ$

$QS \cong f'(7)\cdot PQ$

$QS \cong \frac{1}{14} \times (0.01)$

So $\sqrt{49.01} \cong 7 + QS = 7 + \frac{0.01}{14}$

2. If $f''(x) = \sin x$, $f'(0) = 7$, and $f(0) = 3$, find $f(x)$.

Since $f''(x) = \sin x$,

Then $f'(x) = -\cos x + C$ 2 for some constant C.

Since $f'(0) = 7$, Then 2

$7 = -\cos 0 + C$, so $7 = -1 + C$, so $C = 8$.

Hence $f'(x) = -\cos x + 8$.

Therefore $f(x) = -\sin x + 8x + D$. 2

Since $f(0) = 3$, Then 2

$3 = -\sin 0 + 8\cdot 0 + D$, so $3 = D$.

So $\boxed{f(x) = -\sin x + 8x + 3.}$ 2