Quiz 6

Name: key

1. Estimate the area under the graph of $f(x) = 1/x$ from $x = 1$ to $x = 3$ using four approximating rectangles and left endpoints.

[7]

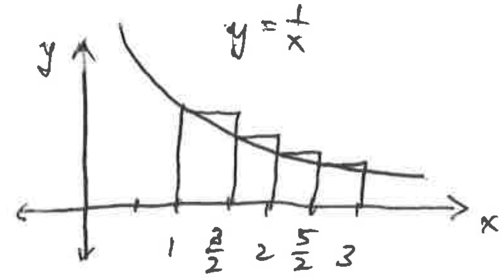
The sum of the areas of the four rectangles (shown in the diagram) is

3

$$\frac{1}{1}\cdot\frac{1}{2} + \frac{1}{(3/2)}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{(5/2)}\cdot\frac{1}{2}$$

which can be simplified to

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{30+20+15+12}{60} = \boxed{\frac{77}{60}}$$



2 Base of each rectangle is $\frac{1}{2}$

2 Height of each rectangle is $y = \frac{1}{x}$ where x is the x-coord of the left side

2. Find the derivative of the function $f(x) = \int_0^{x^2} \sqrt{1+\sin t}\, dt$.

[6]

Let $g(x) = \int_0^x \sqrt{1+\sin t}\, dt$. Then by the Fundamental Theorem of Calculus, $g'(x) = \sqrt{1+\sin x}$. 2

Since $f(x) = g(x^2)$, 2 Then by the Chain Rule,

$$f'(x) = g'(x^2)\cdot 2x = \boxed{\sqrt{1+\sin(x^2)}\cdot 2x}$$ 2

3. Evaluate the integral $\int_1^2 \frac{x+3x^2}{x^4}\, dx$

[7]

2

It equals $\int_1^2 \left(\frac{x}{x^4} + 3\frac{x^2}{x^4}\right)dx = \int_1^2 \left(x^{-3} + 3x^{-2}\right)dx$

2 2

$$= \left[\frac{x^{-2}}{-2} + \frac{3x^{-1}}{-1}\right]_1^2 = \left[\frac{-1}{2x^2} - \frac{3}{x}\right]_1^2 =$$

1

$$= \left[\left(\frac{-1}{8} - \frac{3}{2}\right) - \left(\frac{-1}{2} - \frac{3}{1}\right)\right] =$$

$$= \frac{-1}{8} - \frac{3}{2} + \frac{1}{2} + 3 = \boxed{\frac{15}{8}}$$