Math 1914
Review for Second Exam

The second exam is over sections 2.5, 2.6, 2.8, 3.1, 3.2, 3.3, 3.4, and 3.5 of the text. These sections were covered on Assignments 5 through 9 and Quizzes 3 and 4.

Here is a summary of the topics covered on the test, arranged by section.

2.5 The chain rule. Some people remember the chain rule as the statement that

$$\text{The derivative of } f(g(x)) \text{ is } f'(g(x)) \cdot g'(x).$$

Some remember it as the statement that

$$\text{If } y \text{ is a function of } u, \text{ and } u \text{ is a function of } x, \text{ then } \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

And some people just learn to take the derivative of simple functions using the chain rule, without really ever putting into words what it is they're doing. If you're one of the people in the third category, try to move yourself into one of the first two categories. Knowing the chain rule as an explicit rule will help when you have to make more sophisticated use of it later.

You can do this by taking a function such as $\sin(x^2)$, and writing down carefully what $f(x)$, $g(x)$, $f'(x)$, $g'(x)$, and $f'(g(x))$ are, and then multiplying the last two together. Or write down carefully what y and u are, and compute dy/du and du/dx, and multiply them together to get dy/dx.

You should review the entire section, except you won't need to know how to prove the chain rule (page 153).

2.6. Implicit differentiation. Implicit differentiation is one of the "more sophisticated" uses of the chain rule that I was referring to in the above paragraph. To do it right, you have to be able to, for example, use the chain rule to find the derivative of $\sin(xy)$ with respect to x, or the derivative of $y^2 \dfrac{dy}{dx}$ with respect to x.

You should review the entire section.

2.8. Related rates. Related rates problems all follow the same, very simple, pattern: there are two related quantities involved, say U and V, and you are given the values of U and V and the derivative of one of the quantities with respect to time, and you have to find the derivative of the other quantity with respect to time. For example, you may have that $U^2 + V^2 = 7$, and you are given that $U = 1$ and $dU/dt = 3$, and you have to find dV/dt. What can make the problem tricky is that it might not be immediately obvious from the problem statement what the two quantities are and how they are related. But hopefully this information is there if you look carefully enough!

This section contains several illustrative examples, but the best review would probably be to pick a couple of the exercises at the end that you haven't done yet, and work them out.

3.1. Maximum and minimum values.

You should be clear on the technical meanings of the following terms: absolute maximum and minimum of a function on a set D (box 1 on page 198), local maximum and minimum of a function (box 2 on page 198) (notice the concepts of local maximum and minimum don't refer to a specific set, but the concepts of absolute maximum and minimum do); critical number of a function (box 6 on page 201); increasing and decreasing function on an interval (box on page 19); concave upward and concave downward graphs (box on page 216); and inflection point (first box on page 218). Two important theoretical results are the Extreme Value Theorem (box 3 on page 199) and Fermat's Theorem (box 4 on page 200). Review in particular the "Closed Interval Method" on page 202 and Examples 7 and 8 on page 202.

3.2. The Mean Value Theorem. The Mean Value Theorem turns out to be of great theoretical importance, but for reasons that may not be apparent yet. You should know it for future reference, but you won't be tested on it in this exam. However, I might ask you a problem on Rolle's theorem like the one in Example 2 on page 208, or problems 17 – 22 on page 213.

3.3. How derivatives affect the shape of a graph. Review the entire section. Notice in particular the Increasing/Decreasing Test, the First Derivative Test, the Concavity Test, and the Second Derivative Test in the boxes on pages 214, 215, 217, and 218.

3.4. Limits at infinity; horizontal asymptotes. You should know the definition of horizontal asymptote on page 225; illustrations of how to find horizontal asymptotes are in Examples 2, 3, 4, and 5. Example 11 is also very useful.

Compare the definition of horizontal asymptotes with the definition of vertical asymptotes (see page 57) and slant asymptotes (see the bottom of page 241).

3.5. Summary of curve sketching. This section summarizes what's in the preceding couple of sections. You should review examples 1, 2, 3, and 4; they're very instructive.