Math 1914
Review for Third Exam

The third exam is over sections 2.9, 3.9, 4.1, 4.2, 4.3, and 4.4 of the text. These sections were covered on Assignments 10 through 13 and Quizzes 5 and 6.

Here is a summary of the topics covered on the test, arranged by section.

2.9. Linear Approximations and Differentials. The key formula in this section is formula 1 on page 183,

$$f(x) \approx f(a) + f'(a)(x - a).$$

For example, to use this formula to approximate $\sqrt{9.01}$, you could take $f(x) = \sqrt{x}$ and $a = 9$. Then you can compute $f(a) = 2$ and $f'(a) = 1/6$ easily, and the formula with x replaced by 4.01 tells you that

$$\sqrt{4.01} \approx 2 + \frac{1}{6}(0.01).$$

An alternative notation is to write Δx for $x - a$ and Δy for $f(x) - f(a)$. Then the formula above becomes

$$\Delta y \approx f'(a)\Delta x.$$

Sometimes people use dy to denote $f'(a)\Delta x$, in which case the formula becomes just

$$\Delta y \approx dy.$$

Whichever notation you prefer to use, the formula amounts to the same idea, which can be understood by looking at Figure 5 on page 186: if the horizontal line segment PS in that figure has small length, then the line segments SQ and SR have approximately the same length. The length of the line segment SR is often easy to compute, so this gives us a way to easily approximate SQ.

The examples in this section give several indications of why this might be useful.

3.9. Antiderivatives. You should review the entire section. You should know the antiderivatives in Table 2 on page 270 by heart, but that really doesn't require you to learn anything you don't already know, since this is really just a table of the derivatives you already know, written in reverse. The one formula that looks different from a derivative formulas is that an antiderivative of x^n is $\dfrac{x^{n+1}}{n+1}$. But this is just another way of saying that the derivative of x^n is nx^{n-1}, except that n is replaced by $n + 1$ and both sides divided by $n + 1$.

4.1. Areas and Distances. All that you need to know from this section is how to do problems like the ones assigned in the homework from this section (see also Examples 1 and 4, and problem 1 on Quiz 6).

4.2. The Definite Integral. You should be able to give the definition of a definite integral if asked. Here is an acceptable definition:

Suppose that for each choice of n we split an interval $[a, b]$ into n equal subintervals of length Δx, and choose points x_1^*, x_2^*, ... , x_n^* from each of the intervals. The definite integral of f from a to b is defined to be

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

You can give a definition in your own words if you like, but it should be equivalent to the above definition, and as in the above definition you should describe the meaning of any symbols you use.

You should also be aware of the properties of the definite integral given in the red boxes on pages 303, 304, and 305. Notice that the last of these properties was used in doing a couple of the problems assigned from this section.

You can skip the material in the subsections titled "Evaluating Integrals" and "The Midpoint Rule", from the bottom of page 298 to the top of page 303. I won't ask problems on the exam on this material.

4.3. The Fundamental Theorem of Calculus. You should know both Part 1 and Part 2 of the Fundamental Theorem of Calculus (see the red box on page 317). I won't ask you to prove them, and I won't ask you to cite them verbatim, but you need them to do the problems assigned in this section and section 4.4.

You should review all the examples in this section.

4.4. Indefinite Integrals and the Net Change Theorem. "Indefinite integral" is a synonym for "antiderivative". (Actually, in class we talked about indefinite integrals already when discussing antiderivatives in section 3.9.) So despite the fact that they look similar, $\int_a^b f(x)\,dx$ and $\int f(x)\,dx$ are two fundamentally different things. The first is the limit of Riemann sums, and the second is an antiderivative. Of course, although they are fundamentally different, they are related, and the relation between them is given by the second part of the Fundamental Theorem of Calculus (which relates definite integrals to antiderivatives).

Examples 1 through 6 in this section are worth reviewing as exercises in how to evaluate integrals.

You can skip the material on the "Net Change Theorem" (pages 324 to 326) if you like.