EXAM 2
Math 1914
10-26-15

Name key (white)

Instructions *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(20 points)* Find the derivative, showing all work. (You may use any of the rules of differentiation.)

[10] a. $y = \cos\left(\frac{\sin x}{x}\right)$ $= f(g(x))$ where $f(x) = \cos x$ (1), $f'(x) = -\sin x$ (1), (2)

$g(x) = \frac{\sin x}{x}$ (1), $g'(x) = \frac{x \cdot \cos x - \sin x \cdot 1}{x^2}$,

so $f'(g(x)) = -\sin\left(\frac{\sin x}{x}\right)$ (2), and $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$ (1) $= \boxed{-\sin\left(\frac{\sin x}{x}\right) \cdot \left[\frac{x\cos x - \sin x}{x^2}\right]}$ (2)

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[10] b. $y = \left(\frac{1}{1+\tan(3x)}\right)^5$ $= (1+\tan(3x))^{-5}$ (2), so

$\frac{dy}{dx} = (-5)(1+\tan(3x))^{-6}$ (2) $\cdot \frac{d}{dx}(1+\tan(3x))$ (2)

$= \boxed{\frac{-5}{(1+\tan(3x))^6} \cdot (0 + \sec^2(3x) \cdot 3)}$ (2) (2)

(You could also find $\frac{dy}{dx}$ by using $\frac{dy}{dx} = 5\left(\frac{1}{1+\tan(3x)}\right)^4 \cdot \frac{d}{dx}\left(\frac{1}{1+\tan(3x)}\right) =$ etc.)

2. *(15 points)* Suppose y is a function of x satisfying the equation $3x + 5y + y^3 = 9$. Use implicit differentiation to find:

a. $\frac{dy}{dx}$ when $x = 1$ and $y = 1$.

$\frac{d}{dx}(3x + 5y + y^3) = \frac{d}{dx}(9)$

$\Rightarrow 3 + 5\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0 \Rightarrow 3 + (5 + 3y^2)\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-3}{5+3y^2}$; at $x=1$ and $y=1$ we have $\boxed{\frac{dy}{dx} = \frac{-3}{8}}$

b. $\frac{d^2y}{dx^2}$ when $x = 1$ and $y = 1$.

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{-3}{5+3y^2}\right) = (-3)\frac{d}{dx}\left[(5+3y^2)^{-1}\right] =$

$= (-3)\cdot(-1)[5+3y^2]^{-2} \cdot \left(0 + 6y\frac{dy}{dx}\right) = \frac{3}{(5+3y^2)^2} \cdot 6y \cdot \frac{dy}{dx}$.

So when $y = 1$ and $\frac{dy}{dx} = \frac{-3}{8}$, we have $\frac{d^2y}{dx^2} = \frac{3}{8^2} \cdot 6 \cdot 1\left(\frac{-3}{8}\right) = \boxed{\frac{-54}{8^3}} = \frac{-27}{256}$

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3. *(15 points)* Use linear approximation, and the fact that $\sqrt{1225} = 35$, to estimate $\sqrt{1220}$.

We use $f(x) \approx f(a) + f'(a)\cdot(x-a)$, (2)

with $f(x) = \sqrt{x}$, (2) $x = 1220$, and $a =$ [illegible] 1225. (2)

Then $f(a) = \sqrt{1225} = 35$, (1) and

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$, (2) so $f'(a) = \frac{1}{2\sqrt{a}} = \frac{1}{2\sqrt{1225}} = \frac{1}{2\cdot 35} = \frac{1}{70}$. (1)

Also $x - a = 1220 - 1225 = -5$, so

$\sqrt{1220} \approx 35 + \frac{1}{70}\cdot -5$, (2) (3) or

$\sqrt{1220} \approx 35 - \frac{1}{14} = \boxed{34\frac{13}{14}}$.

4. *(15 points)* Find the maximum and minimum values of $f(x) = x(\sqrt{x} - 3)$ on the interval $[0, 100]$. Show all work.

Maximum value of $f(x)$ is 700 at $x =$ 100

Minimum value is $f(x)$ is -4 at $x =$ 4

$f(x) = x^{3/2} - 3x$, so $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - 3 = \frac{3\sqrt{x}}{2} - 3$. (2)

The critical point(s) are where $\frac{3\sqrt{x}}{2} - 3 = 0$, (2) or $\sqrt{x} = 2$,

or $x = 4$. (2)

Checking $f(x)$ at the endpoints $x = 0$ and $x = 100$, and at the critical point $x = 4$, gives

$$\begin{cases} f(0) = 0\cdot(0 - 3) = 0 & (2) \\ f(4) = 4(\sqrt{4} - 3) = 4(-1) = -4 & (2) \\ f(100) = 100(\sqrt{100} - 3) = 100\cdot 7 = 700, & (2) \end{cases}$$

So the max value is 700 at $x = 100$, and the min value is -4 at $x = 4$. (3)

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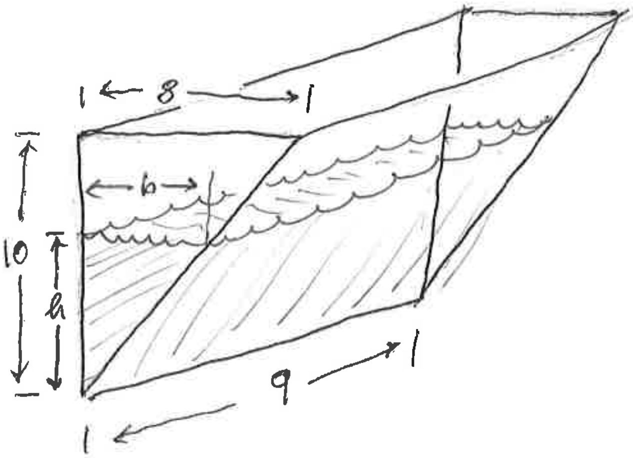
5. *(15 points)* A water trough (see diagram) has a cross-section in the shape of a triangle with base 8 feet and height 10 feet. The trough is 9 feet long. The water in the trough is rising at a rate of $\frac{1}{10}$ feet per minute. How fast is the volume of water in the trough increasing when the water is 5 feet deep?

(Note: the volume of a triangular prism is equal to the product of its length and the area of a cross-section. The area of a triangle is one-half its base times its height.)

Let h be the depth of the water and b be the the distance across the trough at the water's surface. Let V be the volume of water in the trough. (2)



Then $V = \frac{1}{2} bh \cdot 9$ (2)

We are given $\frac{dh}{dt} = \frac{1}{10}\ \frac{\text{ft}}{\text{min}}$ (2), and we want to find $\frac{dV}{dt}$ when $h = 5$.

We have $\frac{dV}{dt} = \frac{d}{dt}\left(\frac{9}{2} bh\right) = \frac{9}{2}\left(b\frac{dh}{dt} + h\frac{db}{dt}\right)$. (2)

From similar triangles, $\frac{b}{h} = \frac{8}{10} = \frac{4}{5}$ (2), so $b = \frac{4h}{5}$.

So $\frac{db}{dt} = \frac{4}{5}\frac{dh}{dt} = \frac{4}{5}\cdot\frac{1}{10} = \frac{2}{25}$. (2)

Also, when $h = 5$, $b = \frac{4}{5}h = 4$. (1) So

$$\frac{dV}{dt} = \frac{9}{2}\left(4 \cdot \frac{1}{10} + 5 \cdot \frac{2}{25}\right) = \frac{9}{2}\left(\frac{2}{5} + \frac{2}{5}\right) = \frac{9}{2}\cdot\frac{4}{5} = \frac{36}{10} = \boxed{\frac{18}{5}\ \frac{\text{cu. ft.}}{\text{min}}}$$

(2)

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6. *(20 points)* For the function $f(x) = 8x^5 - 10x^4$, find the following, showing all work:

a. Critical point(s) $x=0$ and $x=1$

b. Interval(s) of increase $(-\infty, 0)$ and $(1, \infty)$

c. Interval(s) of decrease $(0, 1)$

d. Local maximum(s) $x=0$

e. Local minimum(s) $x = 1$

f ~~a~~. Use the above information to sketch the graph.

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a) $f'(x) = 40x^4 - 40x^3 = 40x^3(x-1) = 0$, so $x=0$ or $x=1$

b)
c)

	$x<0$	0	$0<x<1$	1	$x>1$
sign of x^3 :	−		+		+
sign of $x-1$:	−		−		+
sign of $f'(x)$:	+		−		+
f is:	increasing 2		decreasing 2		increasing 2

d) local max at $x=0$ 2 $\}$ seen from answer to b), c)
e) local min at $x=1$

f)

4

x	y
0	0
1	$8-10=-2$
2	$8\cdot32-10\cdot16$ $256-160=$ big number
-1	$8(-1)-10(1)=-18$