EXAM 3
Math 1914
10-23-15

Name key (white)

Row ____________

Instructions *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(10 points)* Give a definition of the definite integral as a limit of Riemann sums. You should explain the meaning of the symbols you use in your definition.

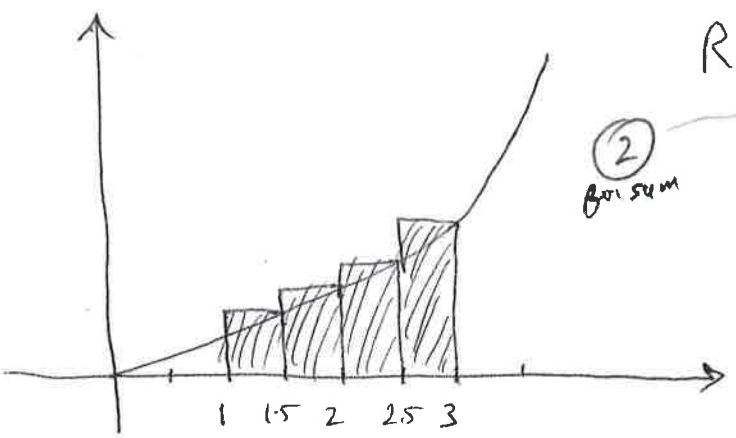
Suppose $f(x)$ is a function (1) defined for x in the interval $[a,b]$. (1) Split $[a,b]$ into n subintervals, each of length Δx, and let x_i (1) denote any point in the i^{th} subinterval, where i ranges from 1 to n. Then

$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

(2) (2) (3)

(provided the limit exists!).

2. *(10 points)* Estimate the area under the graph of $f(x) = x^2$ from $x = 1$ to $x = 3$ using a Riemann sum with 4 approximating rectangles. Use right endpoints.



(2) for sum

$$R_4 = f(1.5)\cdot\tfrac{1}{2} + f(2)\cdot\tfrac{1}{2} + f(2.5)\tfrac{1}{2} + f(3)\cdot\tfrac{1}{2}$$

(2) (1) (1) (1) (1)

$$= \tfrac{1}{2}\left[(1.5)^2 + 2^2 + (2.5)^2 + 3^2\right] \quad (2)$$

$$= \tfrac{1}{2}\left[\tfrac{9}{4} + 4 + \tfrac{25}{4} + 9\right]$$

$$= \frac{86}{8} = \text{[redacted]} = \boxed{\frac{43}{4} = 10.75}$$

3. *(10 points)* Find $\dfrac{d}{dx}\displaystyle\int_0^{\sin x} \sqrt{t^5+1}\,dt$.

Let $f(x) = \int_0^x \sqrt{t^5+1}\,dt$ and $g(x) = \sin x$. (2)

Then we want to find $\frac{d}{dx}(f(g(x)))$, which by the chain rule is $f'(g(x))\cdot g'(x)$. (2) But $f'(x) = \sqrt{x^5+1}$ by the Fundamental Theorem of Calculus, part 1; (2) so $f'(g(x)) = \sqrt{(\sin x)^5+1}$ (2), and $g'(x) = \cos x$.

So the answer is $\boxed{\sqrt{(\sin x)^5+1}\cdot\cos x}$. (2)

4. *(12 points)* Find the area underneath the graph of $y = 4x + \sqrt{x}$, above the line $y = 0$, and between the lines $x = 1$ and $x = 9$.

$$A = \int_1^9 (4x + \sqrt{x})\,dx = \left[2x^2 + \frac{x^{3/2}}{3/2}\right]_1^9$$

$$= \left(2\cdot 9^2 + \tfrac{2}{3}\,9^{3/2}\right) - \left(2\cdot 1^2 + \tfrac{2}{3}\cdot 1^{3/2}\right)$$

$$= \left(2\cdot 81 + \tfrac{2}{3}\cdot 27\right) - \left(2 + \tfrac{2}{3}\right)$$

$$= 162 + 2\cdot 9 - 2 - \tfrac{2}{3} = \boxed{177\tfrac{1}{3}}$$

5. *(25 points)* Evaluate the indefinite integrals, showing all work.

[8] a. $\int x^2\sqrt{1+5x^3}\,dx.$

$$u = 1 + 5x^3$$
$$du = 15x^2\,dx$$
$$\left(\frac{1}{15x^2}\right) du = dx$$

$$= \int x^2 \sqrt{u}\,\frac{1}{15x^2}\,du = \frac{1}{15}\int \sqrt{u}\,du$$

$$= \frac{1}{15}\int u^{\frac{1}{2}}\,du = \frac{1}{15}\,\frac{2}{3}u^{3/2} + C$$

$$= \boxed{\frac{2}{45}(1+5x^3)^{3/2} + C}$$

[8] b. $\int (\sec^2(\sin x))\cos(x)\,dx.$

$$u = \sin x$$
$$du = \cos x\,dx$$

$$= \int \sec^2 u\,du = \tan u + C$$

$$= \boxed{\tan(\sin x) + C}$$

[9] c. $\int \frac{x^3}{(x^2+3)^5}\,dx$

$$u = x^2 + 3$$
$$du = 2x\,dx$$
$$\frac{1}{2x}\,du = dx$$
$$x^2 = u - 3$$

$$= \int \frac{x^3}{u^5}\,\frac{1}{2x}\,du = \int \frac{x^2}{u^5}\,du$$

$$= \int \frac{u-3}{2u^5}\,du = \int \frac{1}{2}\left(\frac{u}{u^5} - \frac{3}{u^5}\right)du$$

$$= \frac{1}{2}\int \left(u^{-4} - 3u^{-5}\right)du$$

$$= \frac{1}{2}\left(\frac{u^{-3}}{-3} - \frac{3u^{-4}}{(-4)}\right) + C$$

$$= \boxed{\frac{-1}{6(x^2+3)^3} + \frac{3}{8(x^2+3)^4} + C}$$

 Name ______________________

6. *(18 points)* For the function $f(x) = \dfrac{3x^2}{x^2 - 16}$, find the following, showing all work:

a. Vertical asymptote(s) $x = 4$ and $x = -4$ 1

b. Horizontal asymptote(s) $y = 3$ 1

c. Critical point(s) $x = 0$ 1

d. Interval(s) of increase $(-\infty, -4)$ and $(-4, 0)$ 2

e. Interval(s) of decrease $(0, 4)$ and $(4, \infty)$ 2

f. Local maximum(s) $x = 0$ 1

g. Local minimum(s) none 1

h. Use the above information to sketch the graph.

a) $x^2 - 16 = 0$ 1 $\quad x = \pm 4$

b) $\displaystyle\lim_{x\to\infty} \frac{3x^2}{x^2-16} = \lim_{x\to\infty} \frac{3}{1 - \frac{16}{x^2}} = 3$ 1

1

c) $\displaystyle y' = \frac{(x^2-16)\cdot 6x - 3x^2 \cdot 2x}{(x^2-16)^2} = \frac{-96x}{(x^2-16)^2} = 0$ 1

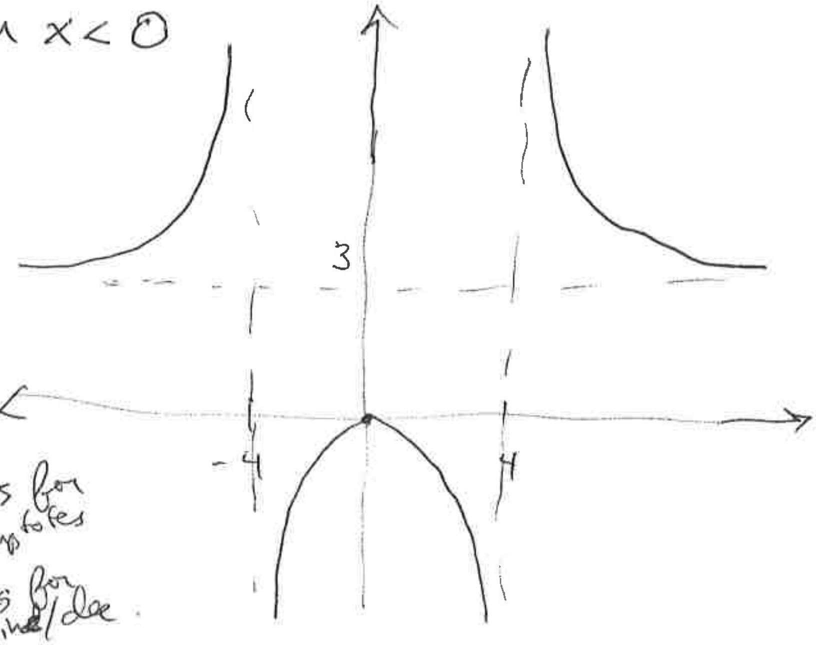
1

$x = 0$

d) $(x^2-16)^2 \geq 0$ for all x in the domain

So y' is negative when $x > 0$

and y' is positive when $x < 0$



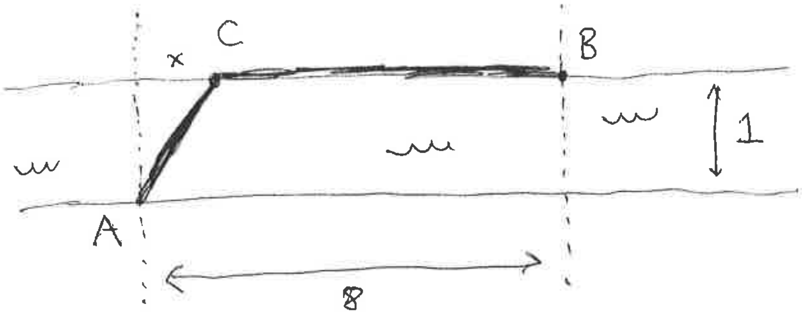
2 pts for asymptotes

2 pts for inc/dec

Name ______________________

7. *(15 points)* You have to lay a cable from point A on one side of a river to point B on the other side, 8 miles down the river. You will do this by laying the cable underwater to a point C on the other side, x miles down the river, and then along the land from C to B. The river is 1 mile wide. (See diagram.)

It costs \$3000 per mile to lay the cable under the water and \$2000 per mile to lay the cable along the land. Find the value of x which minimizes the cost of the cable. Show all work!



Let y be the cost of the cable, in dollars.

Then $y = 3000\cdot AC + 2000\cdot CB$ 2

By the Pythagorean Theorem, $AC^2 = 1^2 + x^2$,

so $AC = \sqrt{x^2+1}$. 2

Also $CB = 8 - x$. 2

So $y = 3000\sqrt{x^2+1} + 2000\cdot(8-x)$.

The minimizing value of x is the solution of

2 1 2

$$\frac{dy}{dx} = 3000\cdot\frac{1}{2}(x^2+1)^{-\frac{1}{2}}\cdot 2x + 2000\cdot(-1) = 0, \text{ or}$$

$$\frac{3000\cdot 2x}{2\sqrt{x^2+1}} = 2000, \text{ or } \frac{3x}{\sqrt{x^2+1}} = 2.$$

So $3x = 2\sqrt{x^2+1}$, 2

$9x^2 = 4(x^2+1)$,

$9x^2 = 4x^2+4$,

$5x^2 = 4$,

$x^2 = \frac{4}{5}$, 2

$$\boxed{x = \sqrt{\frac{4}{5}} = \frac{2\sqrt{5}}{5} \text{ miles}}$$