EXAM 3
Math 1914
10-23-15

Name Key (yellow)

Row

Instructions *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(10 points)* Give a definition of the definite integral as a limit of Riemann sums. You should explain the meaning of the symbols you use in your definition.

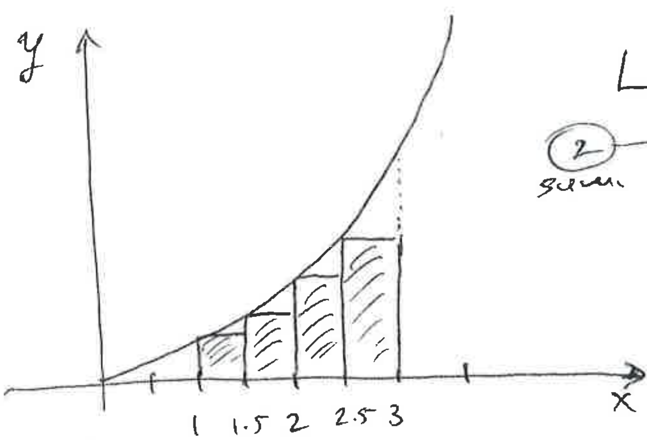
Suppose $f(x)$ is a function defined for x in the interval
Split $[a,b]$ into n 1 subintervals, each of length Δx, 1
and let x_i 1 denote any point in the i^{th} subinterval,
where i ranges from 1 to n. Then

$$\int_a^b f(x)\,dx := \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

2 2 3

(provided the limit exists.)

2. *(10 points)* Estimate the area under the graph of $f(x) = x^2$ from $x = 1$ to $x = 3$ using a Riemann sum with 4 approximating rectangles. Use left endpoints.



2 1 1 1 1

$$L_4 = f(1)\cdot\tfrac{1}{2} + f(1.5)\cdot\tfrac{1}{2} + f(2)\cdot\tfrac{1}{2} + f(2.5)\cdot\tfrac{1}{2}$$

2 sum

$$= \tfrac{1}{2}\left[1^2 + \left(\tfrac{3}{2}\right)^2 + 2^2 + \left(\tfrac{5}{2}\right)^2\right] \quad 2$$

$$= \tfrac{1}{2}\left[1 + \tfrac{9}{4} + 4 + \tfrac{25}{4}\right] = \tfrac{54}{8} = \boxed{\tfrac{27}{4}}$$

3. *(10 points)* Find $\dfrac{d}{dx}\displaystyle\int_0^{x^5} \sqrt{\sin t + 1}\, dt$.

Let $f(x) = \int_0^x \sqrt{\sin t + 1}\,dt$ and $g(x) = x^5$. 2

Then we want to find $\frac{d}{dx}(f(g(x)))$, which by the chain rule is $f'(g(x))\cdot g'(x)$. But $f'(x) = \sqrt{\sin x + 1}$ 2 by the Fundamental Theorem of Calculus, Part 1; and $g'(x) = 5x^4$! 2 So $f'(g(x))\,g'(x) = \boxed{\sqrt{\sin(x^5)+1}\cdot 5x^4}$

4

4. *(12 points)* Find the area underneath the graph of $y = 2x + \sqrt{x}$, above the line $y = 0$, and between the lines $x = 1$ and $x = 9$.

$$A = \int_1^9 (2x + \sqrt{x})\,dx = \left[x^2 + \frac{x^{3/2}}{3/2} \right]_1^9$$

$$= \left[9^2 + \frac{2}{3} \cdot 9^{3/2} \right] - \left[1 + \frac{2}{3} \cdot 1^{3/2} \right]$$

$$= \left[81 + \frac{2}{3} \cdot 27 \right] - \left[1 + \frac{2}{3} \right] = 81 + 18 - 1 - \frac{2}{3} = \boxed{97\tfrac{1}{3}}$$

5. *(25 points)* Evaluate the indefinite integrals, showing all work.

[8] a. $\int x^2 \sqrt{1 + 7x^3}\, dx.$ $= \int x^2 \sqrt{u} \left(\frac{1}{21x^2}\right) du = \frac{1}{21} \int \sqrt{u}\, du$

$$\boxed{\begin{aligned} u &= 1 + 7x^3 \\ du &= 21x^2\,dx \\ \left(\frac{1}{21x^2}\right) du &= dx \end{aligned}}$$

$$= \frac{1}{21} \int u^{\frac{1}{2}}\, du = \frac{1}{21} \frac{2}{3} u^{3/2} + C$$

$$= \boxed{\frac{2}{63} (1 + 7x^3)^{3/2} + C}$$

[8] b. $\int (\cos(\tan x)) \sec^2(x)\, dx.$ $= \int \cos u\, du = \sin u + C$

$$\boxed{\begin{aligned} u &= \tan x \\ du &= \sec^2 x\, dx \end{aligned}}$$

$$= \sin(\tan x) + C$$

[9] c. $\int \frac{x^3}{(x^2 - 2)^5}\, dx$ $= \int \frac{x^3}{u^5} \frac{1}{2x}\, du = \frac{1}{2} \int \frac{x^2}{u^5}\, du$

$$\boxed{\begin{aligned} u &= x^2 - 2 \\ du &= 2x\, dx \\ \frac{1}{2x}\, du &= dx \\ x^2 &= u + 2 \end{aligned}}$$

$$= \frac{1}{2} \int \frac{u+2}{u^5}\, du = \frac{1}{2} \int \left(\frac{u}{u^5} + \frac{2}{u^5} \right) du$$

$$= \frac{1}{2} \int (u^{-4} + 2u^{-5})\, du$$

$$= \frac{u^{-3}}{-6} + \frac{\cancel{2}u^{-4}}{-4} + C$$

$$= \boxed{\frac{-1}{6(x^2-2)^3} - \frac{1}{4(x^2-2)^4} + C}$$

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6. *(18 points)* For the function $f(x) = \dfrac{2x^2}{x^2 - 9}$, find the following, showing all work:

a. Vertical asymptote(s) $x = 3,\ x = -3$ (1)

b. Horizontal asymptote(s) $y = 2$ (1)

c. Critical point(s) $x = 0$ (1)

d. Interval(s) of increase $(-\infty, -3) \cup (-3, 0)$ (2)

e. Interval(s) of decrease $(0, 3) \cup (3, \infty)$ (2)

f. Local maximum(s) $x = 0$ (1)

g. Local minimum(s) none (1)

+9

h. Use the above information to sketch the graph.

(a) $x^2 - 9 = 0$ (1) $x = \pm 3$

(b) $\lim\limits_{x \to \infty} \dfrac{2x^2}{x^2 - 9}$ (1) $= \lim\limits_{x \to \infty} \dfrac{2}{1 - \frac{9}{x^2}} = 2$ (1)

(c) $f'(x) = \dfrac{4x(x^2-9) - 2x^2(2x)}{(x^2-9)^2}$ (1)

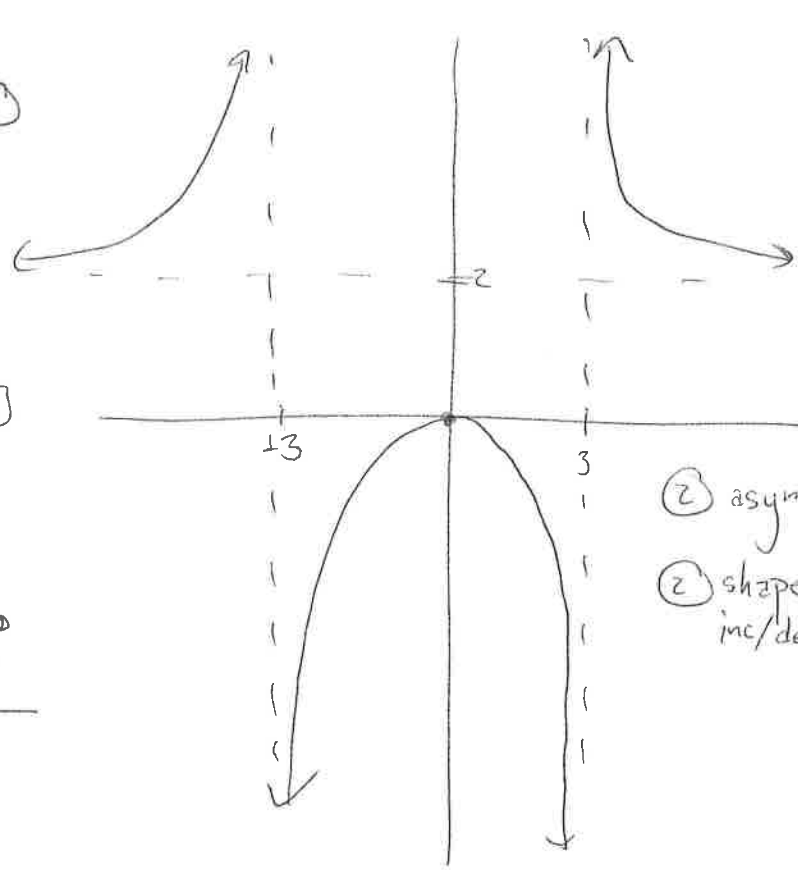
$= \dfrac{4x^3 - 36x - 4x^3}{(x^2-9)^2}$

(1) $= \dfrac{-36x}{(x^2-9)^2} = 0 \Rightarrow \boxed{x = 0}$

(DNE: $x = \pm 3$)

(d)

f	↗	DNE ↗		↘	DNE ↘		
f'	+	DNE +	0	−	DNE	−	
x		−3	0		3		



(2) asymp.

(2) shape/inc/dec.

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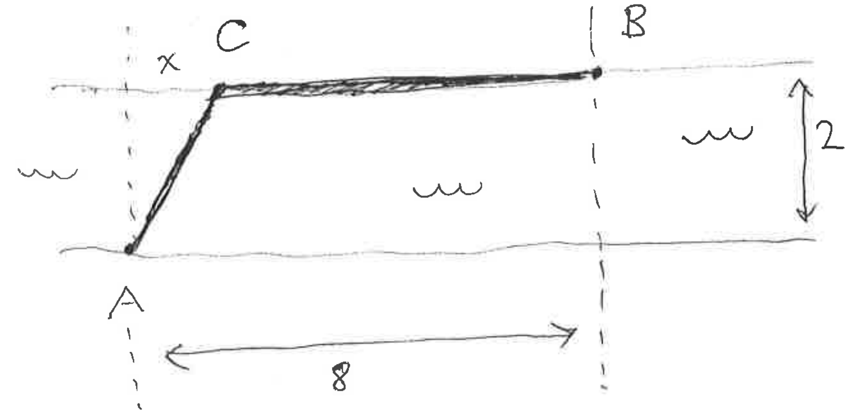
7. *(15 points)* You have to lay a cable from point A on one side of a river to point B on the other side, 8 miles down the river. You will do this by laying the cable underwater to a point C on the other side, x miles down the river, and then along the land from C to B. The river is 2 miles wide. (See diagram.)

   It costs \$3000 per mile to lay the cable under the water and \$1000 per mile to lay the cable along the land. Find the value of x which minimizes the cost of the cable. Show all work!



Let y be the cost of the cable, in dollars.

Then $y = 3000 \cdot AC + 1000 \cdot CB$ 2

By the Pythagorean Theorem, $AC^2 = 2^2 + x^2$,

so $AC = \sqrt{x^2+4}$. 2

Also $CB = 8 - x$. 2

So $y = 3000\sqrt{x^2+4} + 1000\cdot(8-x)$

Set $\frac{dy}{dx} = 3000 \cdot \frac{1}{2}(x^2+4)^{-\frac{1}{2}} \cdot 2x + 1000\cdot(-1) = 0.$ 2 1 2

Then $\frac{3000 \cdot \cancel{2}x}{\cancel{2}\sqrt{x^2+4}} = 1000 \Rightarrow \frac{3x}{\sqrt{x^2+4}} = 1$

$\Rightarrow 3x = \sqrt{x^2+4}$ 2

$\Rightarrow 9x^2 = x^2 + 4 \Rightarrow 8x^2 = 4 \Rightarrow x^2 = \frac{1}{2} \Rightarrow \boxed{x = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}.}$ 2

miles