Quiz 3

Name: key Row: ____________

1. Find the derivatives. You may use any of the rules of differentiation.

[5] **a.** $y = \dfrac{\sin x}{x + \tan x} = \dfrac{u}{v}$

(1) (1)

$$\frac{dy}{dx} = \frac{v u' - u v'}{v^2} = \boxed{\frac{(x+\tan x)\cdot \cos x - \sin x\,(1+\sec^2 x)}{(x+\tan x)^2}}$$

(2) (1)

[5] **b.** $y = x\sin(\sqrt{x}) = u \cdot v$ where $u = x$ and $v = \sin(\sqrt{x})$ (3)

(1) (1)

$$\frac{dy}{dx} = uv' + u'v = x \cdot \frac{d}{dx}\left(\sin(\sqrt{x})\right) + 1 \cdot \sin(\sqrt{x}) = \boxed{x\cos(\sqrt{x})\cdot \tfrac{1}{2}x^{-1/2} + \sin\sqrt{x}}$$

$$\frac{d}{dx}(\sin\sqrt{x}) = f'(g(x))\cdot g'(x) = (\cos\sqrt{x})\cdot \tfrac{1}{2}x^{-\frac{1}{2}}$$

$f(x) = \sin x$ $\quad f'(x) = \cos x$ $\quad f'(g(x)) = \cos\sqrt{x}$

$g(x) = \sqrt{x} = x^{\frac{1}{2}}$ $\quad g'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

2. Suppose y is given implicitly as a function of x by the equation $x^4y^3 + 7x^2 + 11y^2 = 59$.

a. Find dy/dx by implicit differentiation.

[6] $\frac{d}{dx}\left(x^4y^3 + 7x^2 + 11y^2\right) = \frac{d}{dx}(59)$ (1)

(1)

$x^4\frac{d}{dx}(y^3) + \frac{d}{dx}(x^4)y^3 + 7\cdot 2x + 11\cdot\frac{d}{dx}(y^2) = 0$

(1) (1)

$x^4\cdot 3y^2\frac{dy}{dx} + 4x^3y^3 + 14x + 11\cdot 2y\frac{dy}{dx} = 0$

$(3x^4y^2 + 22y)\frac{dy}{dx} = -4x^3y^3 - 14x$ (1)

$$\boxed{\frac{dy}{dx} = \frac{-4x^3y^3 - 14x}{3x^4y^2 + 22y}}$$ (1)

[4] **b.** Find the equation of the tangent line to the graph at the point where $x = 1$ and $y = 2$.

When $x=1$ and $y=2$, $\frac{dy}{dx} = \frac{-4\cdot 1\cdot 8 - 14}{3\cdot 1\cdot 4 + 44} = \frac{-32-14}{12+44} = \frac{-46}{56} = \frac{-23}{28}$

(2)

So the slope of the tangent line is $\frac{-23}{28}$, and its

equation is $\boxed{y - 2 = \left(\frac{-23}{28}\right)(x-1)}$ (2)