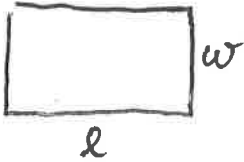


Quiz 4

Name: key Row: _____

[10]

1. The length of a rectangle is increasing at 3 feet per second, and the width is decreasing in such a way that the area stays constant at 10 square feet. How fast is the width decreasing when the length is 5 feet?



$$lw = 10 \quad (1)$$

Find $\frac{dw}{dt}$ when $l = 5$ and $\frac{dl}{dt} = 3$. (1)

$$\frac{d}{dt}(lw) = \frac{d}{dt}(10) \quad (1)$$

$$(2) \quad w \cdot \frac{dl}{dt} + l \cdot \frac{dw}{dt} = 0 \quad (1)$$

when $l = 5$, $w = 2$, so (1)

$$2 \cdot 3 + 5 \cdot \frac{dw}{dt} = 0, \quad \text{so} \quad (1)$$

$$\frac{dw}{dt} = -\frac{6}{5} \text{ ft./sec.} \quad (1)$$

[10]

2. Find the maximum and minimum values of $f(x) = 3x - 4x^3$ on the interval $[0, 2]$. Show all work.

Maximum value of $f(x)$ is 1 at $x = \frac{1}{2}$

Minimum value is $f(x)$ is -26 at $x = 2$

$$f'(x) = 3 - 12x^2 = 0 \quad \text{when} \quad x^2 = \frac{3}{12} \quad \text{or} \quad x = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}.$$

The critical points of $f(x)$ are at $x = -\frac{1}{2}$ and $x = \frac{1}{2}$, but the only one of these in $[0, 2]$ is $x = \frac{1}{2}$. (2) So we check

f at the endpoints $x = 0$ and $x = 2$, and the critical point

$$x = \frac{1}{2}: \quad f(0) = 0 \quad (2)$$

$$f\left(\frac{1}{2}\right) = 3 \cdot \frac{1}{2} - 4 \cdot \frac{1}{8} = 1$$

$$f(2) = 3 \cdot 2 - 4 \cdot 8 = -26$$

So the max is 1 at $x = \frac{1}{2}$
and the min is -26
at $x = 2$

(2)