

Calculus I — Fall 2015 — Review for second exam

The exam covers sections 2.4, 2.5, 2.6, 2.8, 2.9, 3.1, and 3.3 of the text. The relevant homework assignments are Assignments 4, 5, 6, 7, and part of 8.

You should have the following formulas memorized for the exam. (I will not be asking for their proofs, however.)

- The basic rules of differentiation: the sum rule, the product rule, the quotient rule, the power rule, and the chain rule. These are collected conveniently on page 5 of the gray reference pages at the end of the textbook (formulas 1 through 8).
- The derivatives of the basic trig functions: $\sin x$, $\cos x$, $\sec x$, and $\tan x$. (See formulas 13, 14, 15, and 17 on reference page 5.)
- The formula for linear approximation of a function:

$$f(x) \approx f(a) + f'(a)(x - a).$$

See section 2.9.

- As always in calculus, you should know the basic laws of exponents (you should know all the formulas listed in “Exponents and Radicals” on reference page 1 at the end of the text book) and the basic facts concerning trig functions (these are described in the handout “Review of what you need to know about the trigonometric functions” on the course web page, or you can find them listed on reference page 2: you should know everything in the left column on that page and everything in the part labelled “Fundamental identities” on the right.)

Here is a brief guide to the sections covered on the exam. The page numbers refer to the 8th edition; if you own the 7th edition instead, you might want to get hold of an 8th edition for a few minutes to figure out which pages in the 7th edition these refer to.

2.4. Review the entire section. You should know that $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$. I won't ask for the proof of this fact, but you should be able to do homework problems using this fact, like the ones on the assignment from this section. Also, I won't ask for the proof that the derivative of $\sin x$ is $\cos x$ — at least, not on this exam.

2.5. The chain rule is a single formula, but it can be written in two different ways: see the box at the bottom of page 152. In class we mostly stuck with the first way,

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x),$$

and we carefully went through numerous examples of how to apply this formula to specific problems, such as finding the derivative of $\sin(x^2)$, or the derivative of $(\sin x)^2$, by first writing down $f(x)$, then $g(x)$, then $f'(x)$, then $g'(x)$, then $f'(g(x))$, then multiplying the last two together. Be sure you know how to do this! Notice that in this section, the solution to Example 1 is written out the way we did in class (except the author writes $f(u)$ and $f'(u)$ instead of $f(x)$ and $f'(x)$). In the subsequent examples 2 through 7, however, the author does not write out the whole process, but assumes you can do some of the steps in your head.

You should review this entire section, except you can skip the parts concerned with the proof of the chain rule.

2.6. Review the four examples in this section. You can also review the homework problems from this section, and problem 2 on the third quiz. Notice that some problems ask you not only to find dy/dx , but also d^2y/dx^2 using implicit differentiation.

2.8. As mentioned in class, the best strategy for studying related rates problems, or any type of word problem, is to try doing a few of the ones at the end of the section that you haven't tried before. Even if you don't solve the problems, this will help you get the hang of them. Also, of course, you should review carefully the examples in the text.

2.9. Review examples 1 and 3, and the homework problems from this section. You can skip examples 2 and 4, though example 4 is kind of interesting.

3.1. You should know the difference between the absolute maximum on a set of numbers and a local maximum (absolute maxima are always defined with respect to some set of numbers, whereas local maxima are what they are irrespective of any particular set). This difference is illustrated in Example 4. The same goes for minima.

You should know what a critical point is: see definition 6 on page 208. Examples 5 and 6 help to understand what critical points are, and how they are not necessarily the same as maxima or minima, and not necessarily points where the derivative is zero.

Read the box at the top of page 209 on the “Closed Interval Method”. This is often the fastest way to find the absolute maximum and minimum of a function on a closed interval. The method is illustrated in Example 8 and Example 9(b).

The closed interval method works because of “Fermat’s theorem”, which is stated in the box on page 207. You do not need to read through the proof of Fermat’s theorem, but it’s a useful theorem to know.

3.3. You should review the entire section, especially Examples 6 and 7. There will probably not be enough time on the exam to ask you to do a full-fledged graphing exercise like these. But I might ask a shorter question: for example, I may just ask you to find the local maxima and minima, and intervals of increase/decrease of a function.