Quiz 1

Name: key

1. Write down the definitions of the following terms. Briefly explain the meaning of the symbols you use. [6]

a. Riemann sum

Suppose $f(x)$ is a function defined for $x \in [a,b]$, and $[a,b]$ is split into n subintervals of length Δx. Let x_i^* denote a point in the i^{th} subinterval. Then $R_n = \sum_{i=1}^{n} f(x_i^*)\Delta x$ is called a Riemann sum.

b. Definite integral

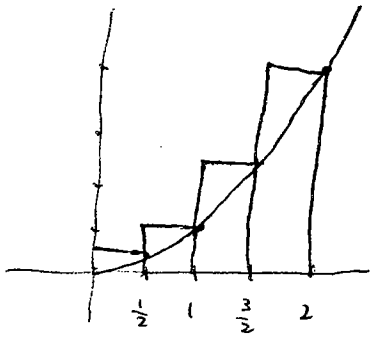
We define $\int_a^b f(x)dx$ to be $\lim_{\substack{n\to\infty \\ \Delta x \to 0}} \sum_{i=1}^{n} f(x_i^*)\Delta x$

2. Evaluate the Riemann sum for $f(x) = x^2$ on $0 \le x \le 2$ using four subintervals and right endpoints. [4]

Each subinterval has length $\Delta x = \frac{1}{2}$. Their right endpoints are $\frac{1}{2}, 1, \frac{3}{2}, 2$; with corresponding values of $y = x^2$ given by $\frac{1}{4}, 1, \frac{9}{4}, 4$. So

$$R_n = \frac{1}{4}\cdot\frac{1}{2} + 1\cdot\frac{1}{2} + \frac{9}{4}\cdot\frac{1}{2} + 4\cdot\frac{1}{2} =$$

$$= \frac{1}{2}\left(\frac{1}{4} + 1 + \frac{9}{4} + 4\right) = \frac{1}{2}\cdot\frac{30}{4} = \boxed{\frac{15}{4}}.$$



3. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function $g(x) = \int_0^{\sin x} \sqrt{1+t^3}\,dt$. [5]

Let $F(u) = \int_0^u \sqrt{1+t^3}\,dt$. Then $g(x) = F(\sin x)$.

So $\frac{dg}{dx} = F'(\sin x)\cdot\cos x$. (1) Since $F'(u) = \sqrt{1+u^3}$, (1) then $F'(\sin x) = \sqrt{1+(\sin x)^3}$ (1)

So $\frac{dg}{dx} = \boxed{\sqrt{1+(\sin x)^3}\cdot\cos x}$ (3)

4. Evaluate the integral $\int_1^2 \frac{x^3+1}{x^2}\,dx$. [5]

$$\int_1^2 \left[\frac{x^3}{x^2} + \frac{1}{x^2}\right]dx = \int_1^2 \left[x + x^{-2}\right]dx = \left[\frac{x^2}{2} + \frac{x^{-1}}{-1}\right]_{x=1}^{x=2} =$$

(1) (1) (1) (1)

$$= \left[\frac{4}{2} + \frac{1/2}{-1}\right] - \left[\frac{1}{2} + \frac{1}{-1}\right] = 2 - \frac{1}{2} - \frac{1}{2} + 1 = \boxed{2}.$$

(1)