**Quiz 6**

Name: key

[10] **1.** Evaluate the integral $\int x^2 \sin(3x)\,dx$. 1 1

$$\int x^2 \sin 3x\,dx = x^2\left(-\frac{\cos 3x}{3}\right) - \int\left(-\frac{\cos 3x}{3}\right)(2x)\,dx$$

1
$$\boxed{\begin{array}{ll} u = x^2 & dv = \sin 3x\,dx \\ du = 2x\,dx & v = -\dfrac{\cos 3x}{3} \end{array}}$$
1

$$= -\frac{x^2\cos 3x}{3} + \frac{2}{3}\int x\cos 3x\,dx$$
1

$$\boxed{\begin{array}{ll} u = x & dv = \cos 3x\,dx \\ du = dx & v = \dfrac{\sin 3x}{3} \end{array}}$$
1 1

1
$$= -\frac{x^2\cos(3x)}{3} + \frac{2}{3}\left[x\,\frac{\sin 3x}{3} - \int \frac{\sin 3x}{3}\,dx\right]$$

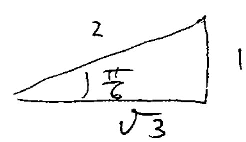
1
$$= -\frac{x^2\cos 3x}{3} + \frac{2}{9}x\sin 3x - \frac{2}{9}\int \sin 3x\,dx$$

1
$$= \boxed{-\frac{x^2\cos 3x}{3} + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C}$$

[10] **2.** Evaluate the integral $\int_0^{\pi/6} \tan^3 x \sec^3 x\,dx$. ~~Simplify your answer as much as possible.~~

Put $u = \sec x$
$du = \sec x \tan x\,dx$. 2

When $x = 0$, $u = \sec 0 = 1$. 2
When $x = \frac{\pi}{6}$, $u = \sec\frac{\pi}{6} = 2/\sqrt{3}$.




So
1
$$\int_0^{\pi/6} \tan^3 x \sec^3 x\,dx = \int_{u=1}^{u=2/\sqrt{3}} \tan^2 x \sec^2 x\,(\sec x \tan x\,dx)$$

$$\boxed{\tan^2 x = \sec^2 x - 1 = u^2 - 1}$$

$$= \int_1^{2/\sqrt{3}} (u^2-1)\cdot u^2\,du \quad 2$$

1 1 1
$$= \int_1^{2/\sqrt{3}} (u^4 - u^2)\,du = \left[\frac{u^5}{5} - \frac{u^3}{3}\right]_1^{2/\sqrt{3}} = \boxed{\frac{1}{5}\left(\frac{2}{\sqrt{3}}\right)^5 - \frac{1}{3}\left(\frac{2}{\sqrt{3}}\right)^3 - \left(\frac{1}{5} - \frac{1}{3}\right)}$$