

Review for Final Exam

For the final exam, which is comprehensive, you can use the review sheets for the first three exams for the material from sections 4.9, 5.1 through 5.5, 6.1 through 6.4, 7.1 through 7.4, 7.6, 7.8, 8.1, 8.2, and 8.3. The definitions and proofs which you should know from these sections are: definition of Riemann sum, definition of "definite integral", proof that the derivative of e^x is e^x ; proof that the derivative of $\ln x$ is $1/x$; proof that the derivative of $\arcsin x$ is $1/\sqrt{1-x^2}$; and proof that the derivative of $\arctan x$ is $1/(1+x^2)$ (see the review sheets for details). Notice that the proofs for the last three derivatives ($\ln x$, $\arcsin x$, and $\arctan x$) all follow the same pattern: to find dy/dx , you first express x in terms of y and find dx/dy , then solve for dy/dx in terms of x .

Besides the sections mentioned above, the final exam will also cover sections 8.4, 8.8, 9.1, and 9.2. (Notice that section 8.7 on approximate integration — the trapezoidal rule and Simpson's rule — is not covered on the exam.)

Here is a brief discussion of what material in the text is relevant to the exam:

8.4. Review from the beginning of the section up to and including Example 6. I will not ask a question in which the integrand has repeated irreducible quadratic factors.

(Note: On the last day of class, the question came up of how to find an integral like $\int \frac{1}{1+\sin x} dx$. Actually, that one is pretty easy, because you can write

$$\begin{aligned}\int \frac{1}{1+\sin x} dx &= \int \frac{1}{1+\sin x} \left(\frac{1-\sin x}{1-\sin x} \right) dx = \int \frac{1-\sin x}{1-\sin^2 x} dx \\ &= \int \frac{1-\sin x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos^2 x} dx,\end{aligned}$$

and each of the latter two integrals is easy (can you do them?).

On the other hand, integrating $\int \frac{1}{2+\sin x} dx$ would not be so easy, because a trick like the one above doesn't work. However, Weierstrass discovered in the 1800's that any rational function of $\sin x$ and $\cos x$ can be converted into an ordinary rational function of u by making the substitution $u = \tan(x/2)$. Then one can integrate the rational function of u using the technique of integration by partial fractions. If you want to see more details about this, you can look at problems 57 to 61 on page 518 of the text. However, this is NOT something that will be covered on the final exam!

8.5 There is no new material in this section; it's just a review of all the integration techniques we've learned up to this point. It would be worth your while to look through this section and try a few of the problems. Also, there's a useful table on page 520 of integration formulas, most of which you should have memorized. (You do not need to memorize formulas number 4, 15, 16, 19, or 20 in this table.)

8.8. Review from the beginning of the section up to and including Example 8. We didn't cover the section on "A Comparison Test for Improper Integrals" (pages 550 – 551) in class, and it won't be covered on the exam.

9.1. Review from the beginning of the section through Example 3. You don't need to read the section on "The Arc Length Function" (pages 564 to 566).

9.2. Review the entire section. You might also take a look at problem 25 on page 573. I won't ask a problem like this on the exam, but it's kind of fun, and educational, to realize that there can be a solid with finite volume and infinite surface area. You can fill it with paint, but you can't paint it!