

Quiz 4

Name: _____

key

Row: _____

[4] 1. Prove that the derivative of $\ln x$ is $1/x$.

Let $y = \ln x$.
 Then $x = e^y$.
 So $\frac{dx}{dy} = e^y$, and $\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$.

2. Differentiate the function:

[4] a. $f(x) = e^{x^2+7x}$
 $g(x) = e^x$ $g'(x) = e^x$ $g'(h(x)) = e^{(x^2+7x)}$
 $h(x) = x^2+7x$ $h'(x) = 2x+7$
 Answer is $g'(h(x)) \cdot h'(x) = e^{(x^2+7x)} \cdot (2x+7)$

[4] b. $f(x) = (\ln x)^5$
 $g(x) = x^5$ $g'(x) = 5x^4$ $g'(h(x)) = 5(\ln x)^4$
 $h(x) = \ln x$ $h'(x) = \frac{1}{x}$
 Answer is $g'(h(x)) \cdot h'(x) = 5(\ln x)^4 \cdot \frac{1}{x}$

3. Evaluate the integral:

[4] a. $\int x^2 e^{x^3} dx$ $= \int e^{x^3} (x^2 dx) = \int e^u \cdot \frac{du}{3}$
 $= \frac{1}{3} \int e^u du$
 $= \frac{1}{3} e^u + C = \frac{1}{3} e^{(x^3)} + C$
 (1) $u = x^3$
 $\frac{du}{dx} = 3x^2$
 $\frac{du}{3} = x^2 dx$

[4] b. $\int \frac{x}{x^2+3} dx$ $= \int \frac{1}{x^2+3} \cdot x dx = \int \frac{1}{u} \cdot \frac{du}{2}$
 $= \frac{1}{2} \int \frac{1}{u} du$
 $= \frac{1}{2} \ln u + C$
 $= \frac{1}{2} \ln(x^2+3) + C$
 (1) $u = x^2+3$
 $\frac{du}{dx} = 2x$
 $\frac{du}{2} = x dx$