Quiz 7

Name: Key Row: _____

Evaluate the integral $\int \frac{7x-1}{x^2+4x-21}\,dx$. Show all work.

$x^2+4x-21 = (x+7)(x-3)$, so write (2)

$$\frac{7x-1}{x^2+4x-21} = \frac{A}{x+7} + \frac{B}{x-3}$$ (2)

Multiplying by $(x+7)(x-3)$ gives

$$7x-1 = A(x-3) + B(x+7)$$ (2)

Put $x=3$: $21-1 = 0 + B\cdot 10 \Rightarrow 20 = 10B \Rightarrow \boxed{B=2}$ (2)

Put $x=-7$: $-49-1 = A\cdot(-10) + 0 \Rightarrow -50 = -10A \Rightarrow \boxed{5=A}$ (2)

So $$\frac{7x-1}{x^2+4x-21} = \frac{5}{x+7} + \frac{2}{x-3}$$, and

$$\int \frac{7x-1}{x^2+4x-21}\,dx = \int \frac{5}{x+7}\,dx + \int \frac{2}{x-3}\,dx$$ (2)

$(u = x+7,\ du = dx)$ $\qquad$ $(w = x-3,\ dw = dx)$

$$= 5\int \frac{1}{u}\,du + 2\int \frac{1}{w}\,dw$$

$$= 5\ln|u| + 2\ln|w| + C = \boxed{5\ln|x+7| + 2\ln|x-3| + C}$$ (3) (3)