Review for Third Exam

For this exam, you should:

• be able to prove that the derivative of $\arcsin x$ is $\dfrac{1}{\sqrt{1-x^2}}$. The proof goes like this:

Let $y = \arcsin x$. Then $x = \sin y$, so $\dfrac{dx}{dy} = \cos y$. Therefore

$$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.$$

• be able to prove that the derivative of $\arctan x$ is $\dfrac{1}{1+x^2}$. The proof goes like this:

Let $y = \arctan x$. Then $x = \tan y$, so $\dfrac{dx}{dy} = \sec^2 y$. Therefore

$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}.$$

The exam will cover sections 6.6, 6.8, 7.1, 7.2, and 7.3 of the text. You should review the problems on Assignments 10, 11, and 12.

Here is a section-by-section guide to the material in the text that will be covered on this test.

6.6 Inverse trigonometric functions. You should review the entire section (and be able to give the two proofs listed at the top of this sheet). You don't need to pay attention to the inverse cosine, cosecant, or cotangent, however, since they won't appear on the test.

This would also be a good time to review the basics of trigonometric functions (see the link to "Review of what you need to know about the trigonometric functions" on the course web page). In particular, you should be familiar with the values of the trigonometric functions at angles which are multiples of 30 degrees or 45 degrees, or in other words angles which are multiples of $\pi/6$ radians or $\pi/4$ radians. You would need to know these if you wanted to find, for example, $\arcsin 1$ or $\arctan -1$. I also recommend memorizing the graphs of $\arcsin x$ and $\arctan x$ (see Figures 4 and 10).

6.8 Indeterminate forms and L'hopital's rule. Review the entire section.

As explained in class, the term "indeterminate form" refers to a type of limit whose value can not be easily determined. It does not mean the same thing as a limit which does not exist, because limits of indeterminate form could actually exist, depending on what functions are actually involved in the limit.

Notice that there several different indeterminate forms mentioned in the section: $0/0$, ∞/∞, $0 \cdot \infty$, $\infty - \infty$, 0^0, ∞^0, and 1^∞; but L'hopital's rule can only be directly applied to limits of the first two types, $0/0$ and ∞/∞. As shown in the examples in this section, you can use L'hopital's rule to help you find limits of the other types as well, but first you have to reduce the problem to finding a limit of one of the first two types.

Also, as mentioned in class, it's useful to know that some limits which look at first like they may be indeterminate are actually not indeterminate. For example, a limit of the form $1/0+$ is not indeterminate, but rather we have that $1/0+ = +\infty$. More precisely, we have that if $f(x)$ and $g(x)$ are functions such that $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = 0$, and $g(x) > 0$ for all x, then $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \infty$.

7.1 Integration by parts. Review this entire section.

7.2 Trigonometric integrals. You should look at all the examples in this section, except you can skip Example 9 on page 500 (and the box above it).

The red boxes on page 497 and page 498 are not meant to be memorized. Rather, they summarize a couple of ideas for evaluating trigonometric integrals which are further explained in the examples. As you read the examples, refer back to the boxes; they'll help you better understand the significance of the examples.

The integrals of $\sec x$ is a bit hard to remember (see box 1 on page 499), so I'll provide it on the exam just in case you need to use it.

7.3 Trigonometric substitution. Review the entire section, except you can skip Example 7. I won't ask problems on this exam like the one in Example 7.

You should memorize the table in the red box at the bottom of page 502, and the triangles that go with it (see Figures 1, 3, and 4). Fortunately, this material is not hard to remember since it's all based on just a couple of trigonometric identities: $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$. And in fact, that is really just one trig identity, since the second one is just the first one divided by $\cos^2\theta$.