## Review for Final Exam

The final exam (Monday, from 1:30 to 3:30, in the usual classroom) is comprehensive, so you should review the material covered on the first three exams, as well as the material we've covered since the third exam. Note that I might ask for the definition of the definite integral, or the definition of $e$, or the proofs of the formulas for the derivatives of $e^x$, $\ln x$, $\arcsin x$, or $\arctan x$.

Here is a guide to reviewing the material from sections 7.4, 7.8, 8.1, and 8.3 of the text (assignments 13 and 14). For the material covered on the first three exams, which also will be covered on the final, you can use the review sheets for those exams.

**7.4 Integration of rational functions by partial fractions.** You should review from the beginning of the section up through Example 5. Notice in particular that Examples 4 and 5 explain what to do in case you have repeated factors in the denominator (meaning that the denominator factors into something like $(x+5)^2(x-2)$, where the factor $(x+5)$ is "repeated"), or in case you have an irreducible quadratic factor in the denominator (meaning that denominator factors into something like $(x-7)(x^2+11)$, where the factor $(x^2+11)$ cannot be further factored into linear factors).

You can skip Examples 6 through 9; I won't be asking problems like those on the final exam.

**7.8 Improper integrals.** Review the entire section.

To summarize the material in this section: there are two types of improper integral. The first type are integrals like $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$, where $\infty$ appears as a limit of integration. These should be computed as limits like $\displaystyle\lim_{b\to\infty}\int_0^b \frac{1}{1+x^2}\,dx$. The second type are integrals like $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$, where the function you are integrating goes to $\infty$ at one end of the interval of integration (in this case, $\frac{1}{\sqrt{x}}$ goes to $\infty$ as $x\to 0$). These should be computed as limits like $\displaystyle\lim_{b\to 0+}\int_b^1 \frac{1}{\sqrt{x}}\,dx$. Sometimes L'hopital's Rule may be useful in evaluating the limits involved.

An improper integral is "convergent" if the limit involved takes a finite value, and "divergent" if the limit is infinite. For example, if you compute them you'll find that $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$ is convergent and $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ is divergent. Sometimes you can tell that an integral is convergent or divergent without actually computing the integral, as in Example 10 (I did another example like this in class April 30th).

Knowing about convergent and divergent improper integrals will be useful in Calculus III, when you study infinite series.

**8.1 Arc Length.** Review from the beginning of the section through Example 2. You should memorize the formula for arc length in Box 3 on page 563. You can skip the material on pages 565–567.

Arc length will come up again in Calculus III and Calculus IV. In fact, the material in section 8.2, which is not covered in this class, will also come up in Calculus III and Calculus IV, so if you are going to take those classes it would be worth your time to have a glance at it now.

**8.3 Applications to physics and engineering.** You should read the subsection on "Moments and centers of mass", which starts at the bottom of page 578, up through Example 6. (You can skip the Theorem of Pappus on page 583, though it's actually quite interesting.) You don't need to memorize the formulas given in this section; I will provide the formulas on the exam if needed.

The material in this section is yet another example of something that will come up repeatedly in Calculus III and Calculus IV. You can see, for example, one of the places it will come up in Calculus IV by looking at pages 1028 to 1030.