FINAL EXAM
Math 2423
5-6-04

Name _____

Row _____

Instructions   *Work all of the following problems in the space provided. If there is not enough room, you may write on the back sides of the pages. Give thorough explanations to receive full credit.*

1. *(10 points)* Give the definition of the definite integral $\int_a^b f(x)\, dx$ as a limit of sums. Remember to explain the meaning of the symbols you use.

2. *(10 points)* Prove that $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$. You may take as given that $\dfrac{d}{dx}(e^x) = e^x$.

3. *(12 points)* Find $\dfrac{d}{dx}\displaystyle\int_1^{x^4} \sqrt{1 + t^3}\, dt$.

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4. *(24 points)* Find the derivative:

a) $\dfrac{d}{dx}\left(\arctan(e^x) \cdot \ln(\sin x)\right)$

b) $\dfrac{d}{dx}\left(x^{(x^3)}\right)$

5. *(12 points)* Find the limit:

a) $\displaystyle\lim_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$

b) $\displaystyle\lim_{x\to\pi} \dfrac{\sin x}{x - \pi}$

6. *(50 points)* Find the indefinite integral, showing all work:

a) $\displaystyle\int \dfrac{dx}{(\arcsin x)\sqrt{1 - x^2}}$

b) $\displaystyle\int xe^{7x}\,dx$

c) $\displaystyle\int \sqrt{1-x^2}\,dx$

   (Hint: remember that $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$, $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$, and $\sin 2\theta = 2\sin\theta\cos\theta$.)

d) $\displaystyle\int \frac{3x-2}{x(x^2+1)}\,dx$

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7. *(14 points)* Evaluate the improper integral $\displaystyle\int_1^\infty \frac{x^2}{(x^3+1)^2}\,dx$.

8. *(16 points)* Find the volume of the solid obtained by rotating the shaded region in the diagram around the y-axis.

9. *(16 points)* A 10-foot long rope, made of material weighing 3 lbs per foot, is lying on the ground. How much work is required to lift one end of the rope to a height of 20 feet?

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10. *(18 points)* Find the area of the surface obtained by rotating the curve $x = \dfrac{y^3}{3}$, $0 \le y \le 1$, around the y-axis.

11. *(18 points)* Find the coordinates $\bar{x}$ and $\bar{y}$ of the centroid of the region shaded in the diagram.