

Review for Test 3

As for Tests 1 and 2, you are encouraged to practice by trying a few of the problems at the end of each section that look similar to the ones that were assigned.

7.4 Derivatives of logarithmic functions (continued). This exam will cover “logarithmic differentiation”, which is described on pages 446–448.

7.5 Inverse trigonometric functions. Read the whole section. I might ask for the proof of the formula for the derivative of the inverse sine function or the formula for the derivative of the inverse tangent function. I gave these proofs in class, and you can also find them in the text. The proof of the formula for the derivative of the inverse sine function starts in the last paragraph on p. 478 and continues to the top of p. 479. The proof of the formula for the derivative of the inverse tangent function is at the top of p. 481. Since you will learn these two proofs for the exam, you’ll also have memorized the formulas for the derivatives of these two functions. I recommend also memorizing the formula for the derivative of the inverse secant function (see the box at the bottom of p. 481). Knowing these three formulas is helpful for doing some of the integrals you see in Chapter 8. You don’t need to know the formulas for the derivative of the inverse cosine, inverse cotangent, or inverse cosecant, however.

7.7 Indeterminate forms and L’hopital’s rule. You should read everything in this section from the beginning up to and including page 498. There are several so-called “indeterminate forms” discussed in this chapter: namely, $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$, 0^∞ , ∞^0 , and 1^∞ . These refer to certain kinds of limits: for example, the limit $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$ would be said to have the form $\frac{0}{0}$, because as x approaches 2, both the numerator and denominator approach 0. On the other hand, the limit $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$ has the form $\infty - \infty$, because as x approaches ∞ , both $\sqrt{x^2 + x}$ and x approach ∞ . The reason for the name “indeterminate” is that limits of this form cannot be evaluated just on the basis of knowing that they are of this form. A limit of the form $\infty - \infty$, for example, could turn out to have the value 0, or ∞ , or even a number like 7. (You might be surprised to know that $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$ turns out to have the value $1/2$.)

The tool we learn to use in this section for evaluating limits of indeterminate form $0/0$ or ∞/∞ is L’hopital’s rule. It is important to notice that it only applies to these two types of indeterminate form. If you are given a limit which has another type of indeterminate form, such as $0 \cdot \infty$, you first have to rewrite the limit in a different form (either $0/0$ or ∞/∞) before you can use L’hopital’s rule. See Examples 6, 7, and 8.

8.1 Integration by parts. Read the entire section, except you can skip Example 6 if you like.

8.2 Trigonometric integrals. I recommend reading the entire section, except for the box at the bottom of p. 523 and the example which follows it (Example 9). However, I do not recommend trying to memorize all the different strategies which are listed in this section. Instead, when reading the solutions for each of the examples, you should just think a bit about why the substitution used in the solution worked, and why other substitutions

do not work. For example, why does the substitution $u = \sec \theta$ work in Example 6, while the substitution $u = \tan \theta$ does not work? After you see the point of these examples, you should be able to look at any similar integral and guess the right substitution, without having to rely on memorizing a long list of strategies.

8.3 Trigonometric substitution. Read the entire section. A better name for this section might be “inverse trigonometric substitution”. In this section, instead of simplifying integrals by making substitutions like $u = \sin x$, where x is the old variable of integration and u is the new one, we simplify integrals by making substitutions like $x = \sin \theta$, where θ is the new variable. Here the new variable is actually an inverse trigonometric function of the old variable; that is, $\theta = \arcsin x$.

It’s easy to recognize when you should try to use an inverse trigonometric substitution to find an integral: inverse trigonometric substitutions are called for when the integral has a quadratic expression (i.e., involving x^2) under a square root sign. You should memorize the “Table of Trigonometric Substitutions” in the box on page 526. Notice that sometimes before making a trigonometric substitution you have to rewrite a quadratic expression by completing the square, as in Example 7. Also notice that many times, after making a trigonometric substitution, you wind up with an integral involving trigonometric functions which resemble the ones in section 8.2. Thus you really should practice the problems in section 8.3 before practicing the ones in 8.3.

8.4 Integration of rational functions by partial fractions. The third exam will only cover the material in this section up through example 4 on pp. 535–6, which was covered on Assignment 11. In other words, only integrals of fractions where the denominator has linear factors will be on this exam. Integrals of fractions where the denominator has irreducible quadratic factors appeared in Assignment 12, and will be covered on the final exam.