Review for Second Exam

For this exam, you should:
• be able to give the definition of e; namely:

$$e = \lim_{h\to 0}(1+h)^{1/h}.$$

There are other definitions of e as well, which you can give if you like. They look different from each other, but they all turn out to define the same number. The definition I've listed here is probably as easy to remember as any of the others. (If you do give another definition, make sure you give a complete one. For example, leaving out the words "e is the number such that ... " in the definition given in the text would *not* result in a complete definition.)

• be able to prove that the derivative of e^x is e^x. The proof goes like this:

$$\begin{aligned}\frac{d}{dx}\, e^x &= \lim_{h\to 0}\frac{e^{x+h}-e^x}{h}\\ &= \lim_{h\to 0}\frac{e^x e^h - e^x}{h}\\ &= \lim_{h\to 0} e^x\left(\frac{e^h-1}{h}\right)\\ &= e^x \lim_{h\to 0}\frac{e^h-1}{h}\\ &= e^x\cdot 1 = e^x.\end{aligned}$$

• be able to prove that the derivative of $\ln x$ is $1/x$. The proof goes like this:
Let $y = \ln x$. Then $x = e^y$, so $\dfrac{dx}{dy} = e^y$. Therefore $\dfrac{dy}{dx} = \dfrac{1}{e^y}$, so $\dfrac{dy}{dx} = \dfrac{1}{x}$.

The exam will cover sections 5.2, 5.3, 5.4, 6.2, 6.3, and 6.4 of the text. You should review the problems on Assignments 5 through 9.

Here is a section-by-section guide to the material in the text that will be covered on this test.

5.2 Volumes. We covered this entire section. In the long run, the easiest way to make sense of the material in this section is to try to fit all the different examples into the single formula given on page 353 ($V = \int A(x)\, dx$). Understanding this formula, and how to use it, is preferable to memorizing a lot of different formulas and different rules for when to apply them.

This formula is called the formula for volume by slices, because $A(x)\, dx$ represents the volume of a thin cross-sectional slice of a solid. The function $A(x)$ is the area of the cross-section of the solid obtained by slicing at location x. The integral $\int_a^b A(x)\, dx$ should be thought of as the result of adding up the volumes of all the slices which comprise the solid.

If the object whose volume is being computed is a solid of revolution around the x-axis, then the cross-sections are either discs, as in Figure 6, or "washers" (discs with holes in them), as in Figure 8. If the object is not a solid of revolution around the x-axis, then the cross-sections will not be circles: they could be squares as in Example 8 or triangles as in Examples 7 and 9.

You should remember that if the slices are perpendicular to the y-axis, then the volume by slices formula becomes $V = \int_c^d A(y)\, dy$ instead. In general, to make sure you are integrating with respect to the correct variable in your volume formula, you should visualize the "elements of volume" you are using — the small pieces you are putting together to make the entire volume — and take the variable of integration to measure the "thin" direction of the element.

5.3 Volumes by cylindrical shells. We also covered this entire section. The method used to compute volumes in this section is different from that used in section 5.2. Here, instead of regarding the volume to be computed as made of lots of slices arranged along the x-axis or the y-axis, we think of it as being made of a lot of thin concentric shells as Figure 4. In the formula in the box on page 364, the expression $2\pi x f(x)\ dx$ represents the volume of one of these shells, and again the integration should be thought of as the process of adding up the volumes of all the different shells.

Notice that in all the figures in this section, the shells are thin in the x-direction, so the figures correspond to integrals with respect to x. In example 3, there is an integral with respect to y, but the shells are not drawn in the accompanying figure. Can you visualize what they would look like?

5.4 Work. Again we covered this entire section. Any question on the test about this material would either be just like Examples 3 or 4, or like Example 5 but with possibly a different shape of cross section (like in problems 21 to 24 at the end of the section). So even if you aren't familiar with the physical notion of "work", you'll be okay if you understand these examples and problems.

6.2 Exponential functions and their derivatives. If you're not quite comfortable with the basics of exponents, please make sure to review these. One way to do this is to Google "review of laws of exponents" and pick a likely-looking site from the results.

You can skip the material on pages 395 and 396, if you like, and replace it with the definition of e given at the top of this sheet, and the proof given at the top of this sheet that the derivative of e^x is e^x. You should also review Examples 2, 3, 5, 7, 8, and 9. You can skip the other examples if you like.

6.3 Logarithmic functions. Section 7.3 covers the basics of logarithms: the definition of logarithm (box 1 on page 404), the laws of logarithms (box 3 on page 404), and the formulas in box 2 on page 404 relating logarithms and exponents. In the text, these are all stated in terms of "logarithms with base a", but on this exam, you will only see natural logarithms, which are logarithms with base e. So in the formulas in the boxes on page 404, you can just replace a by e, and $\log_a$ by ln. (The notation ln means the same as $\log_e$.)

You should review the whole section, except that, since we will only use natural logarithms on this exam, you won't need to know the formula in box 7 at the top of page 407.

6.4 Derivatives of logarithmic functions. You should review all of section 6.4, except that, since there won't be logarithms to other bases than e on the exam, you can skip the box numbered 6 at the top of page 415, and you can skip Example 12.