Quiz 2

Name: key

[7] **1.** A curve is given parametrically by the equations $x = t^3$, $y = 3t^2/2$. Find the length of the curve between the points where $t = 0$ and $t = \sqrt{3}$.

$$L = \int_0^{\sqrt{3}} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^{\sqrt{3}} \sqrt{(3t^2)^2 + (3t)^2}\, dt =$$

$$= \int_0^{\sqrt{3}} \sqrt{9t^4 + 9t^2}\, dt = \int_0^{\sqrt{3}} \sqrt{9t^2}\sqrt{t^2+1}\, dt = \int_0^{\sqrt{3}} 3t\sqrt{t^2+1}\, dt =$$

$$\boxed{\begin{array}{l} u = t^2+1 \\ du = 2t\,dt \end{array}}$$

$$= \int_{u=1}^{u=4} \frac{3}{2}\sqrt{u}\, du = \int_1^4 \frac{3}{2} u^{\frac{1}{2}} du = \left[u^{3/2}\right]_1^4 = 4^{3/2} - 1^{3/2} = 8 - 1 = \boxed{7.}$$

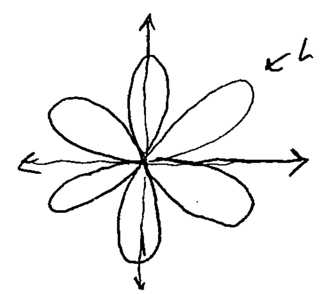
[6] **2.** Find the slope of the tangent to the curve $r = \sin\theta - 1$ at the point where $\theta = 0$. Show all work.

We have $\begin{cases} x = r\cos\theta = \sin\theta\cos\theta - \cos\theta \\ y = r\sin\theta = \sin^2\theta - \sin\theta \end{cases}$, so

$$\left.\begin{cases} \dfrac{dx}{d\theta} = \cos^2\theta - \sin^2\theta + \sin\theta = 1^2 - 0^2 + 0 = 1 \\ \dfrac{dy}{d\theta} = 2\sin\theta\cos\theta - \cos\theta = 0 - 1 = -1 \end{cases}\right\}\text{ when } \theta = 0.$$

Thus when $\theta = 0$, $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{-1}{1} = \boxed{-1.}$

[7] **3.** The graph below shows a six-leaved rose given by $r^2 = 2\sin 3\theta$. Find the area within one loop of the rose.



The loop marked "L_1" corresponds to the part of the graph between $\theta = 0$ and $\theta = \frac{\pi}{3}$. Its area is therefore

$$A = \frac{1}{2}\int_0^{\frac{\pi}{3}} r^2\, d\theta = \frac{1}{2}\int_0^{\pi/3} 2\sin 3\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \sin 3\theta\, d\theta = \left[\frac{-\cos 3\theta}{3}\right]_0^{\frac{\pi}{3}} = \frac{-1}{3}\left[\cos\pi - \cos 0\right]$$

$$= \frac{-1}{3}\left[-1-1\right] = \boxed{\frac{2}{3}.}$$