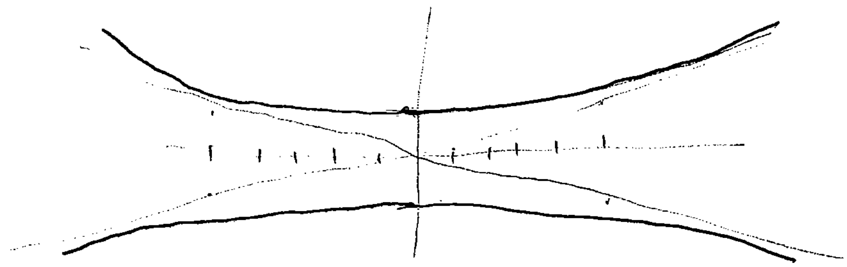
Quiz 3

Name: key

[5] **1.** Sketch the graph of $y^2 - \frac{x^2}{25} = 1$. What are the equations of the asymptotes?



③

Asymptotes $y = \frac{\pm x}{5}$ ②

[4] **2.** Find $\lim_{n\to\infty} \frac{\cos n}{n^2+1}$. Briefly justify your answer.

For all n, $-1 \le \cos n \le 1$, so $\frac{-1}{n^2+1} \le \frac{\cos n}{n^2+1} \le \frac{1}{n^2+1}$. ②

Obviously $\lim_{n\to\infty} \frac{-1}{n^2+1} = 0$ and $\lim_{n\to\infty} \frac{1}{n^2+1} = 0$, so $\lim_{n\to\infty} \frac{\cos n}{n^2+1} = 0$ ② by the Squeeze Theorem.

3. Explain briefly why the series do not converge.

[3] **a.** $\sum_{n=1}^{\infty} \frac{10+n^2}{n^2}$

Here $a_n = \frac{10+n^2}{n^2}$, so $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\frac{10}{n^2}+1}{1} = \frac{0+1}{1} = 1$.

Since a_n does not converge to zero, then $\sum_{n=1}^{\infty} a_n$ does not converge ("n^{th} term test").

[3] **b.** $\sum_{n=1}^{\infty} \frac{10}{n}$

We saw in class that $\sum_{n=1}^{\infty} \frac{1}{n}$ does not converge (because the partial sums are unbounded). The partial sums of $\sum_{n=1}^{\infty} \frac{10}{n}$ are even greater than those of $\sum_{n=1}^{\infty} \frac{1}{n}$, so they're unbounded too; so $\sum_{n=1}^{\infty} \frac{10}{n}$ diverges

[5] **4.** Find $\sum_{n=1}^{\infty} \frac{7}{50^n}$.

We saw in class that if $|r|<1$, then

② $\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}$. Therefore $\sum_{n=1}^{\infty} \left(\frac{1}{50}\right)^n = \frac{\left(\frac{1}{50}\right)}{\left(\frac{49}{50}\right)} = \frac{1}{49}$. ②

So $\sum_{n=1}^{\infty} \frac{7}{50^n} = 7 \cdot \sum_{n=1}^{\infty} \left(\frac{1}{50}\right)^n = \frac{7}{49} = \boxed{\frac{1}{7}}$. ①