Quiz 4

Name: key

1. **Determine whether the series is absolutely convergent, conditionally convergent or divergent. Give reasons for your answers.**

[6] **a.** $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{n^2}}$

$\frac{1}{n^{n^2}} \le \frac{1}{n^2}$ 2, so $\sum \frac{1}{n^{n^2}}$ 2 converges by the comparison test

~~$\frac{1}{n^{n^2}}$ is positive and decreasing ($(n+1)^{(n+1)^2} \ge (n+1)^{n^2} \ge n^2$) and $\lim_{n\to\infty} \frac{1}{n^{n^2}} = 0$, so $\sum \frac{(-1)^n}{n^n}$ converges by the Alt Series test.~~

(not necessary; once we know $\sum \frac{1}{n^{n^2}}$ converges, it automatically follows that $\sum \frac{(-1)^n}{n^{n^2}}$ converges).

So the series is absolutely convergent. 2

[6] **b.** $\sum_{n=1}^{\infty} \frac{n}{(-3)^n}$

Let $a_n = \frac{n}{(-3)^n}$. Then

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| \overset{1}{=} \lim_{n\to\infty} \frac{n+1}{3^{n+1}} \cdot \frac{3^n}{n} \overset{2}{=} \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{1}{3} \overset{1}{=} \frac{1}{3} \overset{1}{<} 1,$$

so the series converges absolutely by the Ratio Test. 1

[8] **c.** $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$

$\sum \frac{1}{n^{1/3}}$ diverges (p-series with $p<1$, 2

or by integral test, because

2 $\int_1^\infty \frac{1}{x^{1/3}}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x^{1/3}}\,dx$

$= \lim_{b\to\infty} \left[\frac{x^{2/3}}{2/3}\right]_1^b = \lim_{b\to\infty} \left(\frac{3b^{2/3}}{2} - \frac{3}{2}\right)$

$= \infty$)

and $\frac{1}{n^{1/3}}$ is positive and decreasing, with $\lim_{n\to\infty} \frac{1}{n^{1/3}} = 0$, 3

so $\sum \frac{(-1)^n}{n^{1/3}}$ converges by Alt. Series test. So the series is 1 conditionally convergent.