

Quiz 7

Name: key

1. Find an equation for the plane containing the point $P(1, 2, 0)$ and perpendicular to the line $x = 2t, y = 1 + t, z = 4 + 3t$.

[6] The vector $\langle 2, 1, 3 \rangle$ is parallel to the line, and so is perpendicular to the plane. (2)

So an equation for the plane is

$$2(x-1) + 1 \cdot (y-2) + 3 \cdot (z-0) = 0 \quad (2)$$

$$\text{or} \quad 2x + y + 3z = 4$$

[6] 2. Find symmetric equations of the line through the points $P(4, 1, 3)$ and $Q(2, -1, 5)$.

A vector parallel to the line is

$$\vec{v} = \vec{PQ} = \langle 2-4, -1-1, 5-3 \rangle = \langle -2, -2, 2 \rangle. \quad (2)$$

So symmetric equations for the line (using P) are

$$\frac{x-4}{-2} = \frac{y-1}{-2} = \frac{z-3}{2} \quad (4)$$

(another form is $4-x = 1-y = z-3$).

[8] 3. Find the point at which the line $x-1 = y = \frac{z}{2}$ intersects the plane $x+y+z = 21$.

Solving the equations of the line for y and z as functions of x gives: $\begin{cases} y = x-1 \\ z = 2x-2 \end{cases} \quad (2)$

(2) The point of intersection must satisfy these two equations as well as the equation of the plane $x+y+z = 21$. Substituting for y and z from above, we get $x + (x-1) + (2x-2) = 21$, or $4x = 24$, so $x = 6$. (2)

Then the equations for the line give $y = 5$ and $z = 10$.

Hence the point is $(6, 5, 10)$. (2)