Quiz 1

Name: ______________________________

1. Suppose $x = t - t^3 + 1$, $y = 2 + t^2$. Find

[5] a) $\dfrac{dy}{dx}$

$\dfrac{dy}{dt} = 2t$ 1 and $\dfrac{dx}{dt} = 1 - 3t^2$ 1, So

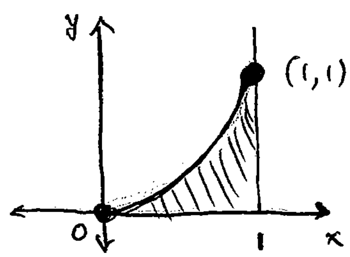
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \boxed{\frac{2t}{1-3t^2}}$$ 2 1

[5] b) $\dfrac{d^2y}{dx^2}$

$$= \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt} = \frac{\frac{d}{dt}\left(\frac{2t}{1-3t^2}\right)}{(1-3t^2)} = \frac{\left[\frac{(1-3t^2)\cdot 2 - 2t(-6t)}{(1-3t^2)^2}\right]}{(1-3t^2)} =$$ 1 1 1 2

$$= \boxed{\frac{(1-3t^2)\cdot 2 - 2t(-6t)}{(1-3t^2)^3}}$$

2. The region shaded in the diagram lies between the x-axis, the line $x = 1$, and the curve $x = \sin t$, $y = 1 - \cos t$, $0 \le t \le \pi/2$. Find the area of the region.



$$\text{Area} = \int_{x=0}^{x=1} y\,dx$$ 2

$$= \int_{t=0}^{t=\frac{\pi}{2}} (1-\cos t)\cos t\,dt$$ 2

$$= \int_0^{\frac{\pi}{2}} (\cos t - \cos^2 t)\,dt$$

$$= \int_0^{\frac{\pi}{2}} \cos t\,dt - \int_0^{\frac{\pi}{2}} \cos^2 t\,dt$$ 1

$$= \left[\sin t\right]_0^{\frac{\pi}{2}} - \left[\frac{1}{2}t + \frac{1}{4}\sin 2t\right]_0^{\frac{\pi}{2}}$$ 1 1

$$= \sin\frac{\pi}{2} - \sin 0 - \left[\left(\frac{1}{2}\cdot\frac{\pi}{2} + \frac{1}{4}\sin\pi\right) - (0+0)\right]$$ 1

$$= \boxed{1 - \pi/4}$$ 2