Quiz 3

Name: answer key

1. Identify the type of conic section given by the equation $x^2 - 6x + 4y^2 + 5 = 0$, and find the vertices and the foci. Sketch the graph of the equation.

[10]

$$x^2 - 6x + 9 + 4y^2 + 5 = 9 \quad (1)$$

$$(x-3)^2 + 4y^2 + 5 = 9 \quad (1)$$

$$(x-3)^2 + 4y^2 = 4 \quad (1)$$

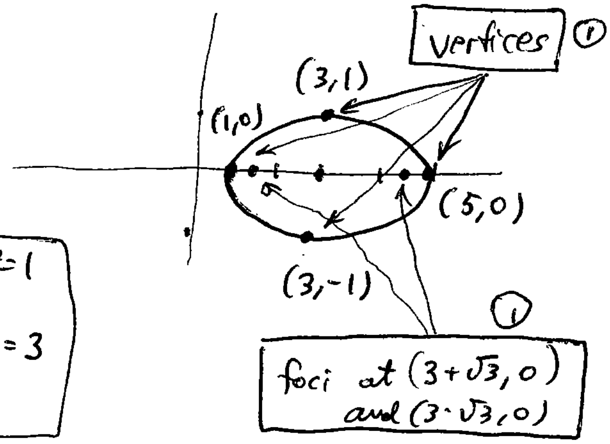
$$\frac{(x-3)^2}{4} + y^2 = 1 \quad (1) \quad \text{(ellipse)} \; (1)$$

$a^2 = 4,\ b^2 = 1$

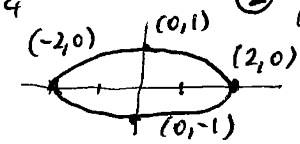
(1)

$c^2 = 4 - 1 = 3$

$c = \sqrt{3}$



The graph of $\frac{x^2}{4} + y^2 = 1$ (2) looks like this:



The graph of $\frac{(x-3)^2}{4} + y^2 = 1$ looks the same, only shifted to the right by 3 units.

2. Determine (with explanation) whether the series is convergent or divergent.

[5] a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3n^2+2}$

Since $\displaystyle\lim_{n\to\infty} \frac{n^2}{3n^2+2} = \lim_{n\to\infty} \frac{1}{3 + \frac{2}{n^2}} = \frac{1}{3+0} = \frac{1}{3} \neq 0$, (1) (1) (1)

Then the series diverges by the Divergence Test. (2)

[5] b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3n^4+2}$

Since $\displaystyle\frac{n^2}{3n^4+2} \le \frac{n^2}{3n^4} = \frac{1}{3n^2}$, (1)

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^2} = \frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n^2}$ is

(1)
convergent (because it is a p-series with $p > 1$) (1), Then $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{3n^4+2}$ (2)
converges by the comparison test.

Alternate problem 2b):

$$\sum_{n=1}^{\infty} \frac{1}{n+8}.$$

(1)

Since $\displaystyle\int_1^{\infty} \frac{1}{x+8}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x+8}\,dx =$

$\displaystyle = \lim_{b\to\infty} \int_9^{b+8} \frac{du}{u}$ (1) $\qquad u = x+8,\ du = dx$

(1)

$\displaystyle = \lim_{b\to\infty} \left[\ln(b+8) - \ln 9\right] = \infty$,

Then the series diverges by the Integral Test. (2)