

Quiz 6

Name: Answer key

[6]

1. Line L_1 is given by the vector equation $\mathbf{r} = \langle 0, 1, 5 \rangle + t\langle 3, 1, 2 \rangle$, and line L_2 is given by the vector equation $\mathbf{r} = \langle 0, 1, 5 \rangle + t\langle 1, 0, -7 \rangle$. Are the two lines perpendicular to each other? Explain your answer.

Line L_1 is parallel to $\langle 3, 1, 2 \rangle$; and line L_2 is parallel to $\langle 1, 0, -7 \rangle$. Since $\langle 3, 1, 2 \rangle \cdot \langle 1, 0, -7 \rangle = 3 \cdot 1 + 1 \cdot 0 + 2 \cdot (-7) = 3 - 14 = -11 \neq 0$; then $\langle 3, 1, 2 \rangle$ and $\langle 1, 0, -7 \rangle$ are not perpendicular to each other; so the lines are not perpendicular.

[7]

2. Find symmetric equations for the line containing the points $P(1, 0, 3)$ and $Q(6, 1, 5)$.

The line is parallel to the vector $\vec{PQ} = \langle 6-1, 1-0, 5-3 \rangle = \langle 5, 1, 2 \rangle$ and contains the point $P(1, 0, 3)$, so it has symmetric equations

$$\frac{x-1}{5} = \frac{y-0}{1} = \frac{z-3}{2}$$

[7]

3. Find an equation for the plane containing the points $A(2, 4, 5)$, $B(1, 5, 7)$, and $C(-1, 6, 8)$.

The plane contains the vectors $\vec{AB} = \langle -1, 1, 2 \rangle$ and $\vec{AC} = \langle -3, 2, 3 \rangle$, so it is normal to the vector

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix} = \vec{i}(3-4) - \vec{j}(-3+6) + \vec{k}(-2+3)$$

or $\vec{n} = \langle -1, 3, 1 \rangle$; Since the plane contains $A(2, 4, 5)$

it has the equation $(-1)(x-2) + 3(y-4) + 1(z-5) = 0$

$$\text{or } -x - 3y + z = -9$$