Quiz 7

Name: Answer key

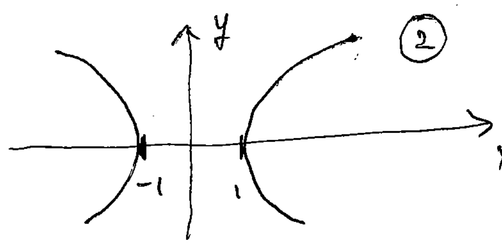
1. Consider the surface in xyz-space defined by the equation $x^2 - y^2 - z^2 = 1$.

[3] a) Does the surface intersect the yz-plane? Explain your answer.

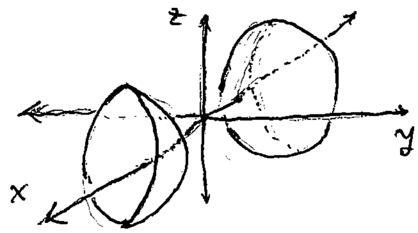
If $x = 0$ then the equation becomes $-y^2 - z^2 = 1$. (1)
There are no solutions of $-y^2 - z^2 = 1$, because $-y^2 - z^2$ must be a negative number. So the surface does not intersect the plane $x = 0$. (2)

[3] b) Sketch the trace of the surface in the xy-plane.

When $z = 0$, the equation becomes $x^2 - y^2 = 1$ (1), which is a hyperbola.



The surface looks like this:



[7] **2.** A surface is given in rectangular coordinates by the equation $z = x^2 + y^2$. Write the equation in spherical coordinates.

Since $x^2 + y^2 = r^2$ (2)
and $r = \rho \sin \phi$ (2)
and $z = \rho \cos \phi$, (2)
The equation $z = x^2 + y^2$
becomes $\rho \cos \phi = (\rho \sin \phi)^2$ (1)

Alternatively, you could use the equations
$x = \rho \sin \phi \cos \theta$ (2)
$y = \rho \sin \phi \sin \theta$ (2)
to write the equation as
$\rho \cos \phi = (\rho \sin \phi \cos \theta)^2 + (\rho \sin \phi \sin \theta)^2$, (2) (1)
which can be simplified to give the same answer as at left.

[7] **3.** Find a vector which is tangent to the curve $\mathbf{r}(t) = \langle t \cos t, \sin t, t \rangle$ at the point $(0, 0, 0)$.

(1) At the point $(0,0,0)$ we have $t = 0$.
So the tangent vector at this point is $\vec{r}\,'(t)$ with $t = 0$.

Now (2) $\vec{r}\,'(t) = \langle \cos t - t \sin t, \cos t, 1 \rangle$, (3)

so when $t = 0$ we have

$$\vec{r}\,'(0) = \langle 1 - 0, 1, 1 \rangle \quad (1)$$

The tangent vector is $\langle 1, 1, 1 \rangle$.

Alternate problem #1 on Quiz 7:

Change $(\sqrt{6}, \frac{\pi}{4}, \sqrt{2})$ from cylindrical coordinates
$\quad\quad\quad\; r \quad\; \theta \quad\; z$

to spherical coordinates (ρ, θ, ϕ).

Solution: Since $x = r\cos\theta = \sqrt{6}\cos\frac{\pi}{4} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$

and $y = r\sin\theta = \sqrt{6}\sin\frac{\pi}{4} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$,

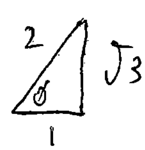
The rectangular coordinates are $x = \sqrt{3}, y = \sqrt{3}, z = \sqrt{2}$.

Therefore $\rho = \sqrt{x^2+y^2+z^2} = \sqrt{3+3+2} = \sqrt{8}$. (3)

We already know $\theta = \frac{\pi}{4}$. (1)

Since $z = \rho\cos\phi$ we have $\sqrt{2} = \sqrt{8}\cos\phi$, or

$\cos\phi = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$, so $\phi = 60° = \frac{\pi}{3}$ radians (2)



So $\boxed{\rho = \sqrt{8},\ \theta = \frac{\pi}{4},\ \phi = \frac{\pi}{3}}$.