Quiz 2

Name: key

1. For the function $f(x,y) = x(e^y + \sin y)$ at the point $P(2,0)$:

a. Find the directional derivative in the direction of the vector $\mathbf{v} = \langle 3, -4 \rangle$. (2)

[5] $\vec{\nabla} f|_P = \langle \frac{\partial f}{\partial x}|_P, \frac{\partial f}{\partial y}|_P \rangle = \langle e^y + \sin y, x(e^y + \cos y) \rangle|_{x=2, y=0} = \langle 1, 4 \rangle$

Unit vector in direction of $\vec{v}$ is $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{\langle 3, -4 \rangle}{\sqrt{3^2+4^2}} = \langle \frac{3}{5}, \frac{-4}{5} \rangle$. (1)

$D_{\vec{u}} f = \vec{\nabla} f|_P \cdot \vec{u} = \langle 1, 4 \rangle \cdot \langle \frac{3}{5}, \frac{-4}{5} \rangle = 1 \cdot \frac{3}{5} - 4 \cdot \frac{-4}{5} = \frac{-13}{5}$ (2)

b. Find the maximum rate of change of f at P, and the direction in which it occurs.

[5] The maximum rate of change is in the direction of $\vec{\nabla} f|_P = \langle 1, 4 \rangle$ (2), and it equals $\|\vec{\nabla} f|_P\| = \sqrt{1^2+4^2}$ (2) $= \sqrt{17}$. (1)

2. Find the equation of the tangent plane to the surface $x^2 + 2xy - y^2 + z^2 = 7$ at the point $(1, -1, 3)$.

[5] Let $f(x,y,z) = x^2 + 2xy - y^2 + z^2$. The surface is a level surface for f, so it is normal to $\vec{\nabla} f|_P = \langle 2x+2y, 2x-2y, 2z \rangle|_P = \langle 0, 4, 6 \rangle$. (2) The tangent plane is normal to $\langle 0, 4, 6 \rangle$ and passes thru $P(1,-1,3)$, so has equation $0(x-1) + 4(y+1) + 6(z-3) = 0$. (3)

3. Find the critical point of the function $f(x,y) = x^2 + xy + 3x + 2y + 5$. (You do not need to say what kind of critical point it is.)

[5] At the critical point, $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$. (1) So x, y satisfy $\begin{cases} 2x + y + 3 = 0 \\ x + 2 = 0 \end{cases}$ (2). Solving gives $x = -2$ and $y = 1$. The critical point is $(-2, 1)$. (2)