Quiz 3

Name: key

[6] **1.** Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the rectangle $R = \{(x, y) : -1 \le x \le 1, -1 \le y \le 1\}$.

$$\int_{-1}^{1}\int_{-1}^{1}(x^2+y^2)\,dx\,dy = \int_{-1}^{1}\left[\frac{x^3}{3}+xy^2\right]_{x=-1}^{x=1}dy = \int_{-1}^{1}\left[\left(\frac{1}{3}+y^2\right)-\left(\frac{-1}{3}-y^2\right)\right]dy$$

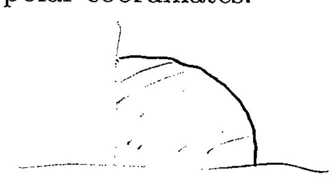
$$= \int_{-1}^{1}\left(\frac{2}{3}+2y^2\right)dy = \left[\frac{2y}{3}+\frac{2y^3}{3}\right]_{-1}^{1} = \left(\frac{2}{3}+\frac{2}{3}\right)-\left(-\frac{2}{3}-\frac{2}{3}\right) = \frac{8}{3}.$$

[7] **2.** Find the integral of $f(x, y) = x/y$ over the region in the first quadrant bounded by the lines $y = x$, $y = 2x$, $x = 1$, and $x = 2$ (see figure).

$$\int_{1}^{2}\int_{x}^{2x}\frac{x}{y}\,dy\,dx = \int_{1}^{2}\left[x\ln y\right]_{y=x}^{y=2x}dx = \int_{1}^{2}x\left[\ln(2x)-\ln(x)\right]dx$$

$$= \int_{1}^{2}x\ln 2\,dx = \left[\frac{x^2\ln 2}{2}\right]_{1}^{2} = \frac{4}{2}\ln 2-\frac{1}{2}\ln 2 = \boxed{\frac{3}{2}\ln 2}.$$

[7] **3.** Evaluate the integral $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}}(x^2+y^2)\,dx\,dy$ by converting to polar coordinates.



$$= \int_0^{\frac{\pi}{2}}\int_0^{1} r^2\,r\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{1} r^3\,dr\,d\theta = \int_0^{\frac{\pi}{2}}\left[\frac{r^4}{4}\right]_0^1 d\theta = \frac{1}{4}\int_0^{\frac{\pi}{2}}d\theta = \frac{1}{4}\cdot\frac{\pi}{2} = \frac{\pi}{8}.$$