Quiz 4

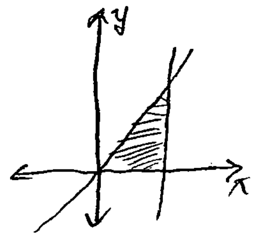
Name: key

[7] **1.** A thin plate of density 1 and thickness T covers the region in the first quadrant between the x-axis, the line ~~x~~ $X = 1$, and the line $y = 2x$ (see diagram). Find the moments of inertia:

a. I_x

$$= \int_0^1 \int_0^{2x} y^2 \, dy \, dx \cdot T = \int_0^1 \left[\frac{y^3}{3}\right]_0^{2x} dx \cdot T$$

$$= T \int_0^1 \frac{8x^3}{3} \, dx = T\left[\frac{2x^4}{3}\right]_0^1 = \boxed{\frac{2}{3} \cdot T}$$



b. I_y

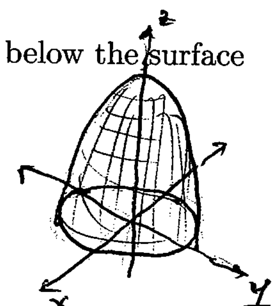
$$= \int_0^1 \int_0^{2x} x^2 \, dy \, dx \cdot T = T \cdot \int_0^1 \left[x^2 y\right]_{y=0}^{y=2x} dx$$

$$= T \int_0^1 2x^3 \, dx = T\left[\frac{x^4}{2}\right]_0^1 = \boxed{\frac{1}{2} \cdot T}$$

c. I_0

$$= I_x + I_y = \frac{2}{3}T + \frac{1}{2} \cdot T = \boxed{\frac{7}{6} T}$$

[6] **2.** Find the volume of the region in xyz-space above the unit circle and below the surface $z = 4 - 4(x^2 + y^2)$ (see diagram).



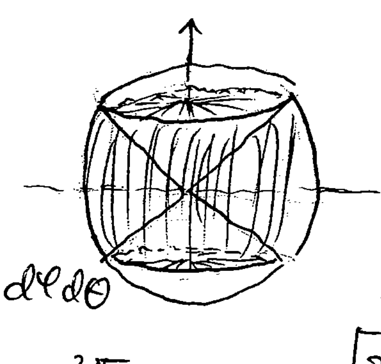
$$V = \iint_{\text{unit circle}} \int_0^{4-4(x^2+y^2)} dz \, dx \, dy$$

$$= \int_0^{2\pi} \int_0^1 \int_0^{4-4r^2} dz \, r \, dr \, d\theta = \int_0^{2\pi} \int_0^1 \left[(4-4r^2)\right] r \, dr \, d\theta$$

$$= \int_0^{2\pi} \int_0^1 (4r - 4r^3) \, dr \, d\theta = \int_0^{2\pi} \left[2r^2 - r^4\right]_0^1 d\theta = \int_0^{2\pi} d\theta = \boxed{2\pi}$$

[7] **3.** Find the volume of the portion of the sphere $\rho \leq 1$ that lies between the cones $\phi = \pi/3$ and $\phi = 2\pi/3$ (see diagram).

$$V = \int_{\theta=0}^{\theta=2\pi} \int_{\varphi=\pi/3}^{\varphi=2\pi/3} \int_{\rho=0}^{\rho=1} \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$



$$= \int_0^{2\pi} \int_{\frac{\pi}{3}}^{2\pi/3} \left[\frac{\rho^3}{3} \sin\varphi\right]_{\rho=0}^{\rho=1} d\varphi \, d\theta = \int_0^{2\pi} \int_{\frac{\pi}{3}}^{2\pi/3} \frac{\sin\varphi}{3} \, d\varphi \, d\theta$$

$$= \int_0^{2\pi} \left[-\frac{\cos\varphi}{3}\right]_{\varphi=\frac{\pi}{3}}^{\varphi=2\pi/3} d\theta = \int_0^{2\pi} \left[-\frac{\cos\frac{2\pi}{3}}{3} + \frac{\cos\frac{\pi}{3}}{3}\right] d\theta = \int_0^{2\pi} \left[\frac{1}{6} + \frac{1}{6}\right] d\theta = \boxed{\frac{2\pi}{3}}$$