Quiz 5

Name: key

[5] **1.** Evaluate $\int_C (x+y+z)\, ds$, where C is the curve given by $x=t$, $y=1-t$, $z=2t$; $0 \le t \le 1$.

$$\int_0^1 (x+y+z)\sqrt{\left(\tfrac{dx}{dt}\right)^2+\left(\tfrac{dy}{dt}\right)^2+\left(\tfrac{dz}{dt}\right)^2}\, dt \quad (1) =$$

$$= \int_0^1 (t+(1-t)+2t)\sqrt{1^2+(-1)^2+2^2}\, dt \;(1)(1) = \int_0^1 (1+2t)\sqrt{6}\, dt \;(1)$$

$$= \sqrt{6}\,[t+t^2]_0^1 \;(1) = \boxed{2\sqrt{6}}$$

[7] **2.** Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = (3x^2-3x)\mathbf{i} + 3z\mathbf{j} + \mathbf{k}$ and C is the curve given by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$; $0 \le t \le 1$.

$$= \int_0^1 \left\{(3x^2-3x)\frac{dx}{dt} + 3z\frac{dy}{dt} + 1\frac{dz}{dt}\right\} dt \;(1) \quad \text{where } \begin{cases} x = t \\ y = t^2 \\ z = t^4 \end{cases} (1);$$

$$= \int_0^1 \left\{(3t^2-3t)\cdot 1 + 3t^4\cdot 2t + 1\cdot 4t^3\right\} dt \;(2)$$

$$= \int_0^1 \left\{3t^2-3t+6t^5+4t^3\right\} dt = \left[t^3 - \frac{3t^2}{2} + t^6 + t^4\right]_0^1 \;(2)$$

$$= 3 - 3/2 = \boxed{3/2}. \;(1)$$

3. Suppose $\mathbf{F} = 2xy\mathbf{i} + (x^2-z^2)\mathbf{j} - 2yz\mathbf{k}$.

[4] **a.** Find a function f such that $\mathbf{F} = \nabla f$.

$$\frac{\partial f}{\partial x} = 2xy \Rightarrow f = x^2y + C(y,z) \;(1)$$

$$\frac{\partial f}{\partial y} = x^2 - z^2 \Rightarrow 2xy + \frac{\partial C}{\partial y} = x^2 - z^2 \Rightarrow \frac{\partial C}{\partial y} = -z^2 \Rightarrow C = -yz^2 + D(z) \;(1)$$

$$\Rightarrow (1)\; f = x^2y - yz^2 + D(z)$$

$$\frac{\partial f}{\partial z} = -2yz \Rightarrow -2yz + \frac{\partial D}{\partial z} = -2yz \Rightarrow \frac{\partial D}{\partial z} = 0 \Rightarrow D = \text{constant} \;(1)$$

So $\boxed{f = x^2y - yz^2 + \text{constant}}$

[4] **b.** Evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where C is any curve which starts at $(3,1,0)$ and ends at $(2,1,-1)$.

By Fund Th of Line Integrals,

$$\int_C \vec{F}\cdot d\vec{r} = f(2,1,-1) - f(3,1,0) = \;(2)$$

$$= (2^2\cdot 1 - 1(-1)^2) - (3^2\cdot 1 - 1\cdot 0^2) = 4 - 1 - 9 = \boxed{-6} \;(2)$$