Quiz 7

Name: key

1. A surface is parametrized by the equations

$$x = \cos u, \quad y = \sin u, \quad z = v$$

for $0 \le u \le 2\pi$ and $0 \le v \le 3$. Find:

(3) **a.** $\mathbf{r}(u,v)$ $= (\cos u)\,\vec{i} + (\sin u)\,\vec{j} + v\,\vec{k}$ $\quad (= x\vec{i} + y\vec{j} + z\vec{k})$

(3) **b.** $\mathbf{r}_u$ $= (-\sin u)\,\vec{i} + (\cos u)\,\vec{j} + 0\,\vec{k}$

(3) **c.** $\mathbf{r}_v$ $= 0\,\vec{i} + 0\,\vec{j} + 1\,\vec{k}$

(3) **d.** $\mathbf{r}_u \times \mathbf{r}_v$ $= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin u & \cos u & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos u - 0)\,\vec{i} - (-\sin u - 0)\,\vec{j} + 0\,\vec{k}$

$= \cos u\,\vec{i} + \sin u\,\vec{j} + 0\,\vec{k}$

(3) **e.** $|\mathbf{r}_u \times \mathbf{r}_v|$ $= \sqrt{\cos^2 u + \sin^2 u + 0^2} = \sqrt{1} = 1.$

(4) **f.** $\int\int_S \mathbf{F} \cdot d\mathbf{S}$, where $F = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$

$$= \int_0^{2\pi}\int_0^3 (x\vec{i} + y\vec{j} + z^2\vec{k}) \cdot (\vec{r}_u \times \vec{r}_v)\, dv\, du$$

$$= \int_0^{2\pi}\int_0^3 (\cos u\,\vec{i} + \sin u\,\vec{j} + v^2\vec{k}) \cdot (\cos u\,\vec{i} + \sin u\,\vec{j} + 0\,\vec{k})\, dv\, du =$$

$$= \int_0^{2\pi}\int_0^3 (\cos^2 u + \sin^2 u)\, dv\, du$$

$$= \int_0^{2\pi}\int_0^3 1\, dv\, du$$

$$= \int_0^{2\pi} 3\, du = \boxed{6\pi}$$

(2) **g.** Sketch a picture of the surface in xyz-space: