Review for Second Exam

The second exam will cover sections 16.1 through 16.8 of the text.

16.1. You should know how a double integral is defined as the limit of Riemann sums. Not that you need to know this in order to compute double integrals (although computer programs for computing double integrals do use Riemann sums). Rather, understanding the integral as a sum makes it easier to understand why integrals are used to find volumes, mass, moments, etc. It will also help in understanding the material in Chapter 17.

I recommend reading the first four pages of this section, up through Example 1 on page 990. You can skip the remainder of the section.

16.2, 16.3. These sections explain how to compute integrals over regions in the plane by writing them as iterated integrals. You should review both sections in their entirety, but the main ideas are all contained in Example 2 on page 1004. You should be very clear on the process of determining the limits of integration in an iterated integral over a region R. If the integral is in the order $dy\ dx$, you first draw a typical vertical line and find the values of y (as functions of x) at which it enters and leaves R, then find the smallest and largest values of x within R. If the integral is in the order $dx\ dy$, you draw a typical horizontal line and find the values of x (as functions of y) at which it enters and leaves the region, and then find the smallest and largest values of y within R.

Example 3 on pp. 1004–1005 makes clear that sometimes the choice of whether to use $dy\ dx$ or $dx\ dy$ makes a big difference in how complicated the integral looks.

16.4. We covered the whole section. You should memorize Figure 2, from which you can read off the formulas relating polar coordinates to x and y; and Figure 5, which explains why the formula for an element of area in polar coordinates is $dA = r\ dr\ d\theta$.

16.5. We covered all the material in this section from the beginning up until just before equation 9 on page 1020. You can skip the part after that — we did not cover radius of gyration, or the subsections on probability or expected values.

16.6. You should read this entire section. You needn't worry too much, however, about what "type 1", "type 2", and "type 3" regions are. This particular way of classifying regions isn't important in itself. What the author is doing here is merely trying to get the method across for determining the limits of the iterated integrals when integrating over a region E in space. It's a natural analogue of what you do for double integrals. For an integral in the order $dx\ dy\ dz$, for example, you start by drawing a typical vertical line and finding the values of z (as functions of x and y) where the line enters and leaves E. Then you look at the projection of E onto the xy-plane, which forms a region R, and draw a typical line through it in the y direction, finding the values of y (as functions of x) where it enters and leaves R. Finally you find the largest and smallest values of x within R.

16.7, 16.8. We covered these sections in their entirety. For both sections, you should familiarize yourself with the coordinate systems first before doing the integration problems.

This means memorizing Figure 2 on page 1037, for cylindrical coordinates, and Figure 5 on page 1042, for spherical coordinates. From these figures you can read off the formulas for changing from one set of coordinates to another: see formulas 1 and 2 on page 1037 and formulas 1 and 2 (and the formulas $z = \rho \cos \phi$ and $z = r \sin \phi$ in the line above those) on page 1042. See also Examples 1 and 2 in section 16.7 and Examples 1 and 2 in section 16.8.

To remember the formulas for the volume element dV in cylindrical and spherical coordinates, you should memorize Figure 7 on page 1038 and Figure 7 on page 1043: they explain the formulas $dV = r\ dr\ d\theta\ dz$ and $dV = \rho^2 \sin \phi\ d\rho\ d\phi\ d\theta$.

With a good feel for cylindrical and spherical coordinates it shouldn't be to hard to set up triple iterated integrals to cover regions E in space. For an integral of the form $\rho^2 \sin \phi\ d\rho\ d\phi\ d\theta$, for example, you would start by drawing a typical line in the ρ direction (straight out from the origin) and find the values of ρ (as functions of ϕ and θ) where the line enters and leaves E. You would then project the entire region E, along lines in the ρ direction, onto the sphere $\rho = 1$, to obtain a region R on the sphere. Then draw a typical line in the longitudinal direction (the direction of ϕ) through R, and find the values of ϕ, as functions of θ, where this line enters and leaves R. Finally you find the largest and smallest values of θ within R.

If the process in that last paragraph struck you as rather a fearsome task, don't worry too much about it for this exam. The triple integrals in spherical coordinates I'd be likely to ask you would be over fairly simple regions, where the most of the boundaries are constant functions of the spherical coordinates.