1.a-

"f is one-to-one" means "if $f(x_1)=f(x_2)$ Then $x_1=x_2$"

To prove this, we assume $f(x_1)=f(x_2)$ and deduce $x_1=x_2$.

For $f(x)=3x+2$:

$3x_1+2 = 3x_2+2$ (premise)

$3x_1 = 3x_2$ (subtract 2 from both sides)

$x_1 = x_2$ (div. by 3)

This proves that $f(x)=3x+2$ is one-to-one

1b. "$f:A\to B$ is onto" means "for every $y\in B$ There exists $x\in A$ such that $f(x)=y$."

For $f(x)=3x+2$:

(1) ~~fo~~ $y \in \mathbb{R}$ (premise)

(2) Define $x = \frac{y-2}{3}$

(3) $f(x)=3x+2$

(4) $f(x) = 3(\frac{y-2}{3})+2$ (from 2, 3)

(5) $f(x) = (y-2)+2$

(6) $f(x) = y$.

1c. yes, because f is 1-to-1 and onto

1d. for $y\in B$, $f^{-1}(y)$ is defined to be The value of $x\in A$ such that $f(x)=y$ (If $f:A\to B$, Then $f^{-1}:B\to A$)

So for 1b., we see that if $f(x)=3x+2$ $(A=\mathbb{R}, B=\mathbb{R})$, Then $f^{-1}:\mathbb{R}\to\mathbb{R}$ is defined by $f^{-1}(y)=\frac{y-2}{3}$.

Yes, g is one-to-one from $[0,\infty)$ to $[0,\infty)$

2.a. As in 1a, start from "if $g(x_1)=g(x_2)$ and $x_1 \in [0,\infty)$ and $x_2 \in [0,\infty)$" and deduce "$x_1 = x_2$"

$$g(x_1)=g(x_2) \quad \text{and} \quad x_1 \in [0,\infty) \text{ and } x_2 \in [0,\infty).$$

$$x_1^2 = x_2^2 \quad \text{and} \quad x_1 \geq 0 \text{ and } x_2 \geq 0$$

$$\sqrt{(x_1)^2} = \sqrt{(x_2)^2} \quad \text{take square roots.}$$

$$x_1 = x_2 \quad \text{(because if } r \geq 0 \text{ then } \sqrt{r^2}=r\text{)}$$

2b. As in 1b, start from $y \in B$ (here $B = [0,\infty)$) and deduce there exists $x \in A$ s.t. $g(x)=y$.

$y \in B$

$y \geq 0$

Let $x = \sqrt{y}$

Then $x \geq 0$.

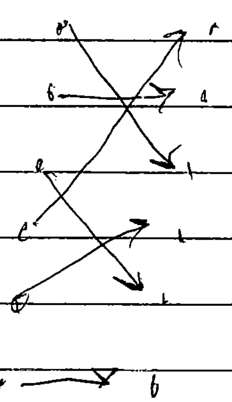
So $x \in A$

$x^2 = y$.

$g(x) = y$.

2c. Yes.

2d. $g^{-1}(y) = \sqrt{y}$.



1e ~~Ans.~~ To show g is not one-to-one, we must show that the statement "if $g(x_1)=g(x_2)$, then $x_1=x_2$" is false. For this, it's enough to find a counterexample.
We have to find values of x_1 and x_2 such that if $g(x_1)=g(x_2)$, then $x_1=x_2$

For $g(x)=x^2$, Take $x_1=-1$ and $x_2=1$. Then $x_1^2=1$ and $x_2^2=1$, so $x_1^2=x_2^2$ so $g(x_1)=g(x_2)$. But $x_1 \neq x_2$. So g is not one-~~to~~-one.

1f ~~Ans.~~ ~~To show g is not onto, we must that the statement "for $y \in [0,\infty)$~~

To show g is not ~~one-to-one~~ onto, we must show that the statement ~~iff~~ "for every $y \in \mathbb{R}$, there exists $x \in \mathbb{R}$ such that $x^2=y$" is false. For this it enough to find a counterexample

Take $y=-1$. For every $x \in \mathbb{R}$, $x^2 \geq 0$, so $x^2 \neq -1$. So there is no $x \in \mathbb{R}$ such that $x^2=y$. This shows $g: \mathbb{R} \to \mathbb{R}$ is not onto.

1g ~~Ans.~~ No, because not one-to-one (and not onto, either)

1h ~~Ans.~~ Does not apply.

1i. $f \circ g(x) = f(g(x)) = 3\,g(x)+2 = 3x^2+2$

(or $f \circ g(x) = f(g(x)) = f(x^2) = 3x^2+2$)

1j. $g \circ f(x) = g(f(x)) = \cancel{g}\,(f(x))^2 = (3x+2)^2$

(or $g \circ f(x) = g(f(x)) = g(3x+2) = (3x+2)^2$.)