Math 2513
Review for First Exam
May 28, 2013

The first exam will be on the material that we've covered in class through Tuesday, from chapters 1, 2 and 4; this material was also covered on Assignments 1 through 6. Here is a more specific discussion of what portions of the text will be covered on the exam.

1.1. Propositional Logic. Propositions and compound propositions. Negations ($\neg$), conjunctions ($\wedge$), disjunctions ($\vee$), conditional statements ($\rightarrow$), biconditional statements ($\leftrightarrow$). Truth tables for compound propositions. Review pages 1 through 10. Note in particular the terms "contrapositive" and "converse", as described in Example 9. We use these terms frequently in this course. You won't need to know the material on pages 11 to 12 about bit strings.

1.2. Applications of Propositional Logic. Review the subsections titled "Translating English Sentences" on pages 16 to 17, and "Logic Puzzles" on pages 19 to 20. You can skip the rest of this section.

1.3. Propositional Equivalences. This section describes the notions of equivalence between propositions (denoted by $\equiv$). You should also be familiar with the meanings given in Definition 1 on page 25 of the terms "tautology" (symbolized by **T**) and "contradiction" (symbolized by **F**). Review the material in this section from the beginning through page 30. You can skip the last two subsections on "Propositional Satisfiability" and "Applications of Satisfiability".

In particular, you should be aware of the Logical Equivalences in Tables 6, 7, and 8. You do not need to memorize all these, but it would be worth memorizing at least De Morgan's Laws and the Distributive Laws from Table 6 on page 27. Also, you should notice the parallel between the identities in Table 6 and the set identities in section 2.2 (see Table 1 on page 130). They are really the same table, with the only difference being that $\wedge$ in one table plays the role of $\cap$ in the other table, $\vee$ in one table plays the role of $\cup$ in the other table, and $\neg$ in one table plays the role of the operation of taking the complement (signified by the bar over the set name) in the other table. The reason that the two tables are the same is that they are actually two instances of one and the same abstract mathematical structure, known as a "Boolean algebra", which you can read more about on Wikipedia or in Chapter 12 of our text. By the way, you might get some benefit out of trying to figure out what statements about sets correspond to the logical equivalences in Tables 7 and 8 on page 28.

1.4. Predicates and Quantifiers. You should know the meanings of the terms "propositional function", "domain of discourse", "universal quantifier" ($\forall$), "counterexample", and "existential quantifier" ($\exists$). You should review from the beginning of the section through page 49. Pay special attention to De Morgan's laws for quantifiers in Table 2. You can skip the material on pages 50 to 52, but you probably would get some benefit by reading through the examples from Lewis Carroll.

1.5. Nested Quantifiers. This important section should be reviewed in its entirety. You can skip Example 16 if you like, but if you ever take the class Intro. to Analysis I, you'll find this example very useful.

1.6. Rules of Inference. Definition 1 in this section is very important. Whenever a homework problem or test problem in this class asks you to "prove" or "show" something, it is asking you to construct a valid argument, in the sense of this definition. That is, a valid argument or proof has to have clearly stated premises and conclusions, and the conclusions have to be deducible from the premises by a sequence of propositions, each of which can be shown to be always true whenever the statements preceding it are true. In deducing statements from other statements you can use rules of inference such as the ones in Table 1 on page 72. You do not need to memorize these, but I would suggest at least reading through them, and memorizing one or two such as modus ponens and modus tollens. We will not be constructing a lot of formal proofs in this course, but knowing the basics about how formal proofs are constructed will go a long way towards helping you to learn the more colloquial style used in writing proofs in this and other upper-division math courses, such as Intro. to Analysis or Higher Algebra.

Review the whole section, except you can skip the subsection titled "Resolution", and you don't need to dwell on the rules of inference for quantified statements (pages 75 to 78). Again, though, reading how formal proofs are constructed for quantified statements will be a big help in future courses where you have to write proofs of quantified statements.

1.7. Introduction to Proofs. You should know what proofs by contraposition and proofs by contradiction are. I suggest reading a couple of examples from this section of each of these types of proof. The problems on the test will hopefully be fairly similar to the ones on the homework, so a good way to practice might be to pick out homework problems from the end of the section that look like the ones that were assigned, and try to do them.

1.8. Proof Methods and Strategy. The main topic we covered from this section is "Proof by Cases". You should read a couple of the examples on pages 92 through 95. We did Example 5 in class, and a similar problem on the homework from this section. It's not necessary to read the remainder of the section (pages 97 on), though there are a number of interesting proofs in there.

2.1. Sets. The entire section should be reviewed carefully. Notice that, by definition, two sets A and B are equal if both of the statements "if $x \in A$ then $x \in B$" and "if $x \in B$ then $x \in A$" are true. So one way to prove the equality of the two sets is to prove these two statements.

2.2. Set operations. Review the section from the beginning up through Example 15. Pay particular attention to Examples 10 through 14, which illustrate several different methods for proving set identities. Definitions 6 and 7 and Examples 16 and 17 are also interesting. You can skip the subsection on "Computer Representation of Sets".

2.3. Functions. Review this section from the beginning through page 147. You should also know the difference between a function and a partial function, as defined on page 152. Remember that the definition of function that we worked with in class was not the informal one given in the blue box on page 139, but rather the set-theoretic definition given in the paragraph just above Definition 2 on page 139, and explained more fully in class. You should know this definition, along with the meanings of the terms "domain", "codomain", and "range" (see Definition 2). Notice that the codomain is not the same as the range, in general. Also know the meanings of the terms "one-to-one" and "onto" as applied to functions (synonymous with "injective" and "surjective").

You can skip the material on pages 148 through 151.

2.4. Sequences and summations. Review from the beginning through example 10, and the subsection on "Summations" from the beginning (at the bottom of page 162) through Theorem 1. You can skip Example 20 and Examples 21 through 25.

2.5. Cardinality of sets. Although I did discuss some of the material of this section in class, it will not be covered on the exam.

4.1. Divisibility and Modular Arithmetic. You should review the entire section. On the first exam, I will not ask for proofs involving the material in this section, but you should be able to do calculations like those in problems 21, 27, and 46.