Math 2513
Review for Second Exam
June 13, 2014

The second exam will cover sections 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 5.1, 5.2, and 5.3 of the text. The problems will be similar to those done in class and on homework assignments 6 through 11.

4.3 Direct Proof and Counterexample III: Divisibility. In this section are the definition of "n is divisible by d" or "$d|n$", and a couple of basic and important theorems on divisibility: the fact that every integer greater than 1 is divisible by some prime number, and the Fundamental Theorem of Arithmetic (Theorem 4.3.5). You should know these. Interestingly, we did not fully prove either of these theorems when we were discussing this section in class. That's because the proofs are best done using induction (actually strong induction, in the case of the Fundamental Theorem of Arithmetic), so we put them off until we got to Chapter 5, where induction is discussed. You won't need to know the proofs for the exam.

4.4 Direct Proof and Counterexample III: Division into Cases and the Quotient-Remainder Theorem. You should memorize the statement of the Quotient-Remainder Theorem (Theorem 4.4.1). This is the fact that underlies the definition of the expressions n *div* d and n *mod* d; which you should also know. You should also know the definition of the absolute value of a number, given on page 187. Again, we didn't prove the quotient-remainder theorem here, because the proof is best done using induction. This section contains several examples of the method of proof by division into cases, which you should look at to see how such proofs are properly written.

4.5 Direct Proof and Counterexample IV: Floor and Ceiling. You should know the definition of the floor function, given on page 191. It's worth reviewing Examples 4.5.4 and 4.5.5 on pages 191–193. You can skip the last two pages of the section, though they're not hard to read.

4.6 Indirect Argument: Contradiction and Contraposition. You should be able to supply a proof by contradiction or a proof by contraposition of a statement, if asked. Actually, there is not a big distinction between these two types of proofs, as is noted in the discussion on page 203. Let's put it this way. If you're asked prove a statement by contradiction, then you should start by negating the statement, then make that negated statement an assumption and proceed from there to somehow get a contradiction. If you're asked to prove a statement by contraposition, the statement should be put (if it isn't already) in the form of a universal conditional statement $\forall x \in D, P(x) \rightarrow Q(x)$; then you should rewrite it as an equivalent universal statement in which the conditional is replaced by its contrapositive, $\forall x \in D, \sim Q(x) \rightarrow \sim P(x)$; then prove the latter statement by assuming $\sim Q(x)$ (for a generic x in D) and deducing $\sim P(x)$. The difference between the two methods is illustrated in the text by giving two different proofs of Proposition 4.6.4, one by contraposition at the bottom of p. 202, and one by contradiction at the bottom of page 203.

4.7 Indirect Argument: Two Classical Theorems. The two classic theorems proved here (Theorems 4.7.1 and 4.7.4) have very beautiful and simple proofs, which are good illustrations of proofs by contradiction. Since the theorems are on the same topics that are covered in the other sections of chapter 4 (divisibility and primes), it would help your understanding for if you took just a few minutes to re-read their proofs.

4.8 Algorithms. The only part of this section we covered was the material on pages 220 to 223, on the Euclidean algorithm. You can skip the rest of the section.

5.1 Sequences. You should know the definition of "sequence", given on page 228. We didn't actually talk much in class about the rest of this section, because it's pretty elementary. I think you know most of it already, and whatever you don't know you can read for yourself easily. One thing we have discussed in class, though, is the recursive definition of summation, given right after Example 5.1.8 on page 232. (You can also find this recursive definition of summation in Chapter 5 on page 300.) We've also mentioned the recursive definition of n factorial, which you can find on page 237. You can skip the rest of the material in this section, for now.

5.2 Mathematical Induction I. We covered pretty much this entire section in detail.

5.3 Mathematical Induction II. You should review Examples 5.3.1 and 5.3.2; we've also done examples similar to these in class.