Quiz 2 — Math 2934

Name: key

[10]

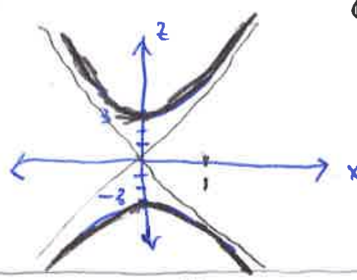
1. For the surface given by the equation $9x^2 + 36y^2 - z^2 + 9 = 0$:

a) Sketch the trace (if any) in the xz-plane.

$y = 0 \rightarrow 9x^2 - z^2 + 9 = 0$

(1) $\rightarrow x^2 - \frac{z^2}{9} = -1$

(2)



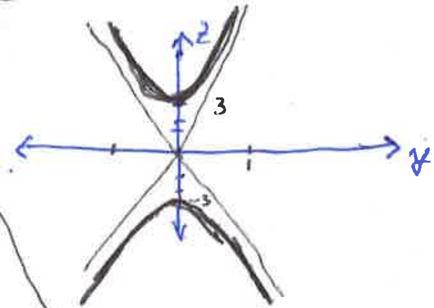
(hyperbola with asymptotes $z = \pm 3x$) and z-intercepts $z = \pm 3$

b) Sketch the trace (if any) in the yz-plane.

$x = 0 \rightarrow 36y^2 - z^2 + 9 = 0$

(1) $4y^2 - \frac{z^2}{9} = -1$

(2)



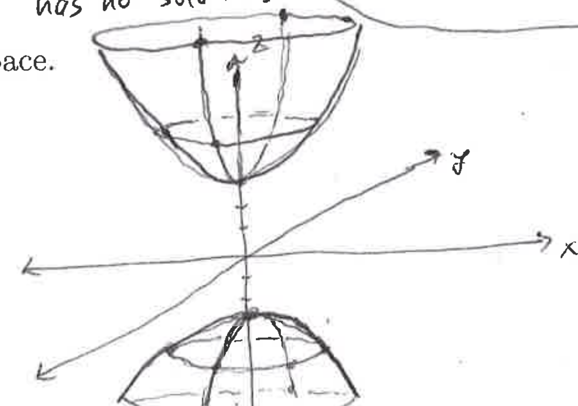
(hyperbola with asymptotes $z = \pm 6x$) and z-intercepts $z = \pm 3$.

c) Sketch the trace (if any) in the xy-plane.

There is no trace in the xy-plane, because for $z = 0$, the equation (2) $9x^2 + 36y^2 = -9$ has no solutions.

d) Sketch the surface in space.

(2)



(It's a hyperboloid of two sheets. Traces in planes parallel to the xy-plane are ellipses.)

[10]

2. The curve C is the intersection of the surface $z = x^2 + 4y^2$ with the [redacted] plane $y = x$.

a) Give parametric equations for C.

(2) Let $x = t$. Since C lies in the plane $y = x$, then (2) $y = t$ also. And then since C lies in the surface $z = x^2 + 4y^2$, we have

$z = t^2 + 4t^2$, or $z = 5t^2$ (2) along C. So

$$\begin{cases} x = t \\ y = t \\ z = 5t^2 \end{cases}$$

b) Find a vector tangent to C at the point $(3, 3, 45)$.

At this point, $t = 3$, and C is tangent to (2) $\left\langle \frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt} \right\rangle \Big|_{t=3}$

(2) $= \langle 1, 1, 10t \rangle \big|_{t=3}$

$= \boxed{\langle 1, 1, 30 \rangle}$.

(or any vector parallel to that).